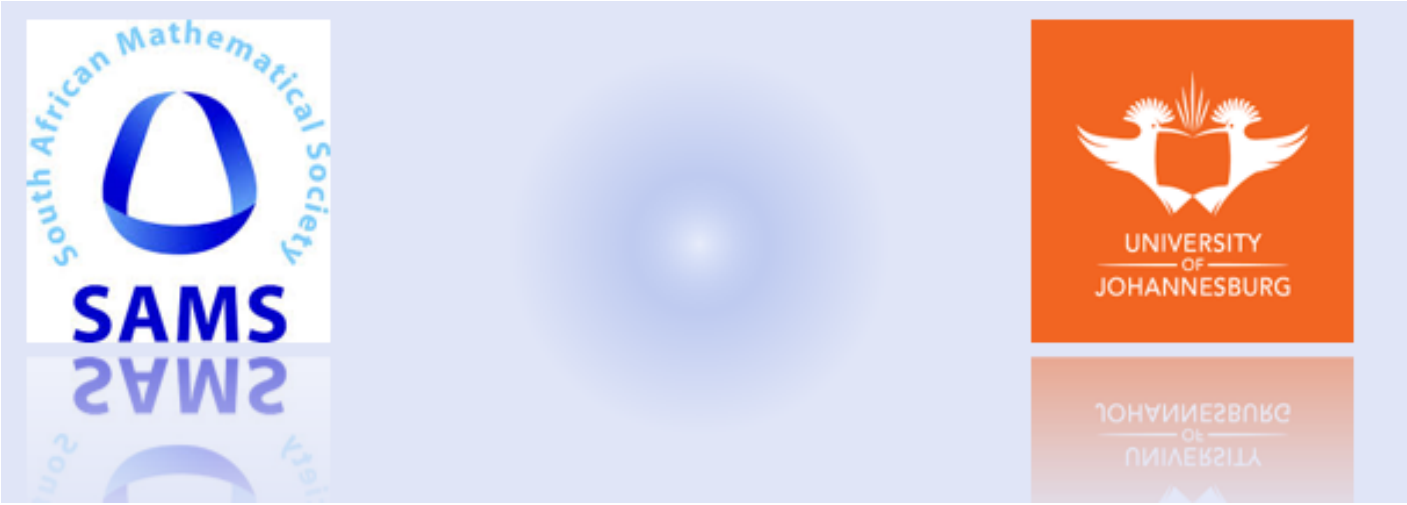
# The 66th Annual Congress of the South African Mathematical Society (SAMS)

University of Johannesburg

BOOK OF ABSTRACTS




**5 – 7 December 2023**




# Welcome to the SAMS Annual Congress 2023

The Department of Mathematics and Applied Mathematics of the University of Johannesburg is pleased to welcome all of the delegates to the 66th SAMS Annual Congress.

We gratefully acknowledge financial support from the following organizations:

The South African Mathematical Society,
The National Institute for Theoretical and Computational Sciences,
The National Graduate Academy: Mathematical and Statistical Sciences,
The National Research Foundation,
The DSI-NRF Centre of Excellence in Mathematical and Statistical Sciences.

We trust that all delegates will enjoy the talks and also engaging with fellow mathematicians.

# Information on the cocktail function at Constitutional Hill

The Tuesday evening cocktail function will take place at Constitution Hill in Braamfontein. This is the site of the Constitutional Court, built on the site of a former prison. Buses will be available to transport delegates to and from the venue. If you are driving, there is secure parking at Constitution Hill. Buses will return delegates to the UJ Bunting Rd campus at the end of the function. As soon as the last talks on Tuesday are finished we ask delegates to make their way to the buses, which will leave from outside where the lunch is served.

### Constitutional Hill Directions

**Step 1:** Take Bunting Rd to Annet Rd approximately 6 min (1.2 km)

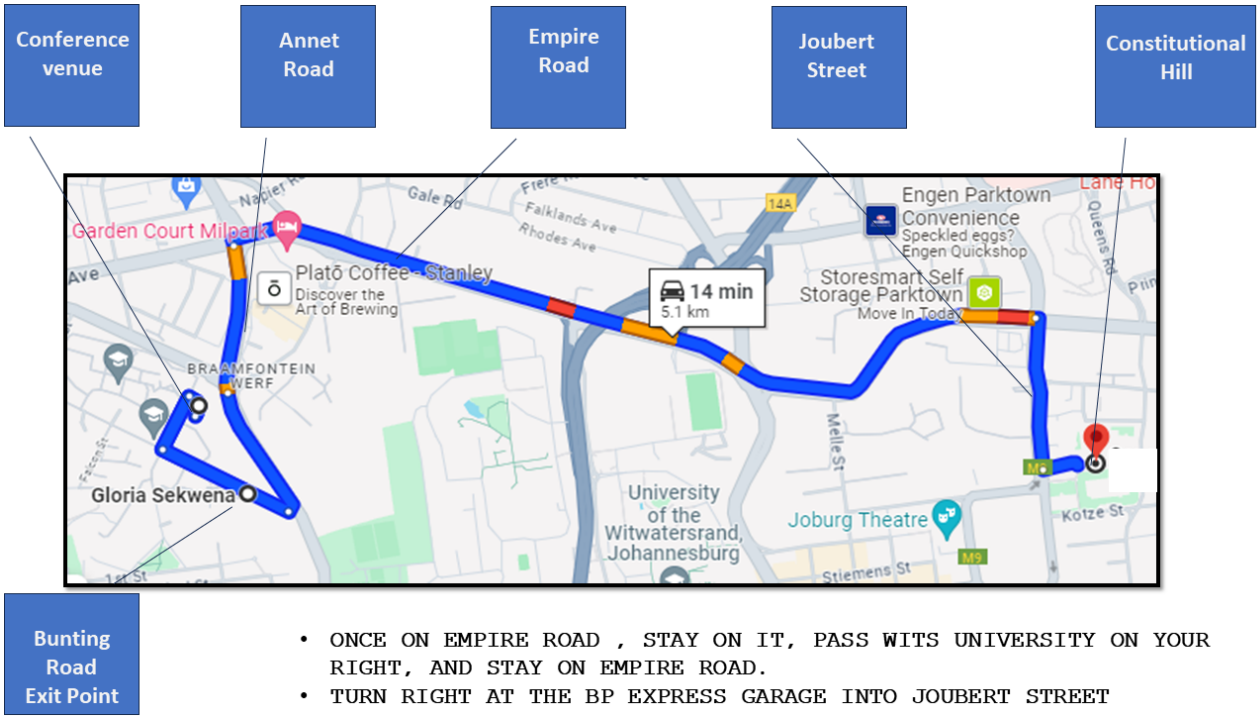
**Step 2:** Drive along Empire Rd approximately 9 min (3.8 km)

**Step 3:** At BP Express service station, turn right into Joubert Street.

**Final Step:** Turn left into the Constitution Hill Human Rights Precinct after passing the first left turn.

**Parking:** For those not taking the bus/shuttle there is underground parking which they usually let guests use. From there use the lift to go to the main building. It is important to inform the security personnel that you are attending a SAMS function.

### Map




Directions to Constitutional Hill

# SAMS 2023 PROGRAMME

| | | | |
|---|---|---|---|
| CATL - Category Theory, Algebra, Topology and Logic | Mathematics Education | General | Differential Equations |
| Finite Groups | FAOT - Functional Analysis and Operator Theory | Lie Symmetries | Graph Theory and Number Theory |

| Day 1 | Plenary Venue | Venue 1 | Venue 2 | Venue 3 | Venue 4 | Venue 5 | Venue 6 | Venue 7 |
|---|---|---|---|---|---|---|---|---|
| 08:00 - 08:30 | Opening | | | | | | | |
| 08:30 - 09:20 | B Rodrigues - On two-weight codes invariant under finite simple groups. Chair: A Basheer | | | | | | | |
| Chair | A Goswami | | | S Madanha | JH van der Walt | E Andriantiana | F Chirove | S Mbusi |
| 09:30 - 09:55 | G Janelidze | | | H Saydi | L Labuschagne | P Dankelmann | M Nchabeleng | D Mason |
| 09:55 - 10:20 | G Boxall | | | M Devi | C Pretorius | E Rivett-Carnac | A Ngana | C Khalique |
| 10:20 - 10:50 | Coffee | | | | | | | |
| 10:50 - 11:15 | W Conradie | | | C Mokalapa | M Hassen | C Mennen | A Tass´e | G Magalakwe |
| 11:15 - 11:40 | S Moshokoa | | | M Kekana | F Schulz | F Zottor | B Ngwu | T Motsepa |
| 11:40 - 12:05 | L Nongxa | | | T Nyikadzino | T Lukoto | S Nkonkobe | E Nel | L Moleleki |
| 12:05 - 13:30 | Lunch | | | | | | | |
| 13:30 - 14:20 | NITheCS invited plenary: N Neudauer - Models of Undergraduate Research in Mathematics. Chair: Z Janelidze | | | | | | | |
| 14:20 - 14:50 | Coffee | | | | | | | |
| Chair | J Gray | | | B Rodrigues | E Kikianty | M Sias | F Chirove | D Mason |
| 14:50 - 15:15 | J Raftery | | | S Madanha | S ter Horst | F Tiraga | E Adeyefa | O Adeyemo |
| 15:15 - 15:40 | J Wannenburg | | | A Basheer | M Lasri | O Oyewumi | F Mhlanga | K Plaatjie |
| 15:40 - 16:05 | | | | S Mkiva | J Zeelie | M Henning | G Buzuzi | S Mbusi |
| 16:10 - 18:30 | Cocktail party | | | | | | | |

| Day 2 | Plenary Venue | Venue 1 | Venue 2 | Venue 3 (FG) | Venue 4 | Venue 5 | Venue 6 | Venue 7 |
|---|---|---|---|---|---|---|---|---|
| 08:30 - 09:20 | C Pournara - Senior Phase Mathematics : Do we mind the gap? Chair: R Durandt | | | | | | | |
| Chair | C Rathilal | | | R Durandt | F Schultz | P Dankelmann | F Mhlanga | G Magalakwe |
| 09:30 - 09:55 | Z Janelidze | | | Math Ed Workshop | M Mapaya | T Ncambalala | G Sedebo | G Pai |
| 09:55 - 10:20 | J Gray | | | Math Ed Workshop | D Phelps | Z Shozi | H Magau | A Mehmood |
| 10:20 - 10:50 | Coffee | | | | | | | |
| 10:50 - 11:15 | A Goswami | | | D Molise | JH van der Walt | M Sias | J Bidie | LJ Bodibe |
| 11:15 - 11:40 | A Craig | | | F Ratshisinde | L Naude | E Mrema | J Munganga | B Sebogodi |
| 11:40 - 12:05 | C Robinson | | | M Erasmus | E Zeekoei | E Andriantiana | J Munyakazi | M Mhlongo |
| 12:05 - 13:30 | Lunch | | | | | | | |
| 13:30 - 14:20 | K Jordaan - Orthogonal Polynomials and Symmetric Freud weights. Chair: A Goswami | | | | | | | |
| Chair | J Wannenburg | C Robinson | S Moshokoa | M Erasmus | S ter Horst | Z Shozi | F Mhlanga | C Khalique |
| 14:30 - 14:55 | E Inyangala | M Matlabyana | F Assfaw | R Durandt | S Mukeru | C Kriel | L Rundora | M Lephoko |
| 14:55 - 15:20 | C Msipha | J Passmore | P Djagba | T Seretlo | M van Straaten | R Roux | M Kgarose | C Majola |
| 15:20 - 15:50 | Coffee | | | | | | | |
| 15:50 - 16:15 | W van Amstel | B Iragi | J Rabie | M Sehoana | N Kyakutwika | I Schoeman | M Mkhatshwa | R Kgatle-Maseko |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 16:15 - 16:40 | J Sao Joao | S Mthimkhulu | G Joubert | A Prins | N Nzaganya | J Goedhals | N Nolwazi | Y Gaxela |
| 16:40 - 17:05 | E Theart | T Ngoako | G Axelrod | M Muvungu | G Marewo | K Anderson | | M Mafora |
| 18:00 - 20:30 | Gala Dinner | | | | | | | |

| Day 3 | Plenary Venue | Venue 1 | Venue 2 | Venue 3 | Venue 4 | Venue 5 | Venue 6 | Venue 7 |
|---|---|---|---|---|---|---|---|---|
| 08:30 - 09:20 | M Gabeleh - Extensions of Ky Fan's best approximation theorem using some geometric properties of Banach spaces. Chair: S Moshokoa | | | | | | | |
| Chair | G Boxall | I Naidoo | A Craig | A Swartz | E Zeekoei | N Ralaivaosaona | L Rundora | OD Adeyemo |
| 09:30 - 09:55 | R Ferguson | N Twala | Y Semegni | M Muvungu | Pant | E Adeyefa | S Oloniiju | W Manganye |
| 09:55 - 10:20 | D Mgani | M Nxumalo | D Moore | M Mulaudzi | Makhoshi | F Luca | T Itumeleng | I Mohapi |
| 10:20 - 10:50 | Coffee | | | | | | | |
| 10:50 - 11:15 | K Dayaram | S Dubazana | T Tonisi | S Ali | Owolabi | K Kanapi | V Masunda | |
| 11:15 - 11:40 | R Ferguson | A Avilez | S Corcos | M Pheko | Nashine | K Molapo | W Manamela | |
| 11:40 - 12:05 | N Tshakatumba | B Wessels | M Zweni | M Bime | Kikianty | | S Pedro | |
| 12:05 - 13:30 | Lunch | | | | | | | |
| 13:30 - 14:20 | F Nyabadza - Linking infectious diseases models with data in the presence of dynamic policies. Chair: J Banasiak | | | | | | | |
| 14:30 - 15:30 | SAMS AGM | | | | | | | |
| 15:30 - 15:45 | Coffee | | | | | | | |
| 15:45 - 17:00 | SAMS AGM continues | | | | | | | |
| 17:00 - 17:15 | Break | | | | | | | |
| 17:15 - 19:15 | NITheCS Undergraduate research workshop | | | | | | | |

**5 – 7 December 2023**

# Contents

## On Two-Weight Codes Invariant under Finite Simple Groups

BERNARDO RODRIGUES

*University of Pretoria*

SAMS Classification: 15

A linear $[n, k, d]$-code $C$ is called projective if no two columns of a generator matrix are linearly dependent, i.e., the columns represent pairwise distinct points in a projective $(k-1)$-dimensional space or equivalently, if $d(C^{\perp}) \geq 3$. A code $C$ is called a two-weight code if all nonzero codewords have weight $w_1$ or $w_2$ ($w_1 < w_2$), for some $w_1, w_2$. The dual of a two-weight code belongs to the family of uniformly packed codes, i.e., codes in which the number of codewords at distance 3 from a word $x$ which is at distance 2 from the code is constant, and the number of codewords at distance 3 from a word $x$ which is at distance greater than 2 from the code is also constant. Projective two-weight codes are associated with strongly regular graphs and uniformly packed codes. The class of $2-(v, k, \lambda)$ designs with symmetric difference property (SDP) gives rise to self-complementary codes meeting the Grey–Rankin bound. The set of codewords of minimum weight in a binary linear self-complementary code of even length, meeting the Grey–Rankin bound, constitutes the set of blocks of a quasisymmetric design with the symmetric difference property. Two-weight codes have connections with bent functions and with bent vectorial functions, with divisible codes, self-orthogonal codes, and with secret sharing schemes. The study of two-weight codes remains of great interest in coding theory, for although several infinite families of two-weight codes are known, the problem of their complete classification remains open. This talk will give an introduction to this fascinating interplay by focusing on a small set of examples, chosen mostly for their instructional value. I will illustrate the connections described above with examples of the construction of some $q$-ary two-weight codes and in particular, some binary projective two-weight codes on which finite almost quasisimple groups of sporadic type act transitively as permutation groups of automorphisms. Employing a known construction of strongly regular graphs from projective two-weight codes, we will give examples of new strongly regular graphs invariant under the said groups.

**Brief Bio:** Bernardo Rodrigues is Professor of Mathematics at the University of Pretoria (UP). Bernardo studied at the Pedagogic University Enrique JosÈ Varona in La Habana, Cuba where he earned a Licenciatura in Mathematics and Education (equivalent to an MSc) and moved to the former University of Natal, in Pietermaritzburg, South Africa where he obtained a BSc Honours (1998), MSc in 2000 and a PhD in Mathematics in 2003. Although his early work concerned the theory of finite groups, in particular the study of the extension problem, his current research interests concern with the interplay between Algebraic Coding Theory, Finite Groups, Modular Representation Theory of Finite Groups and Combinatorial Design Theory as well as Computatational Group Theory and Axial Algebras, including both the concrete and the abstract aspects of these subjects.

## Orthogonal Polynomials and Symmetric Freud weights

PETER CLARKSON

*University of Kent*
*p.a.clarkson@kent.ac.uk*

KERSTIN JORDAAN*

*University of South Africa*
*jordakh@unis.ac.za*

ANA LOUREIRO

*University of Kent*
*a.loureiro@kent.ac.uk*

SAMS Classification: 05

In this talk I will present some recent results on polynomials orthogonal with respect to exponential weights on the real line, in particular symmetric Freud weights. I will show that the sequence of generalised higher order Freud weights forms a hierarchy of weights. The associated first moment can be written as a finite partition sum of generalised hypergeometric functions. I will describe properties of the recurrence coefficients in the three-term recurrence relation associated with these orthogonal polynomials. Connection formulae between corresponding sequences of generalised higher order Freud orthogonal polynomials in the framework of Christoffel transformations, where the weight is modified by multiplying with a polynomial, are useful in studying properties of the zeros.

**Brief Bio:** Kerstin Jordaan was a high school mathematics teacher before graduating from the University of the Witwatersrand with a PhD in Mathematics in 2002. She occupied positions at Vista University, University of the Witwatersrand and the University of Pretoria before joining the University of South Africa in 2017 as a full essor in the Department of Decision Sciences. Kerstin has been the part-time executive director of the South African Mathematics Foundation since 2018 and served the academic community as the elected president of the South African Mathematical Society from 2016 to 2019. She is a nominated member of the Academy of Sciences of South Africa and the recipient of a prestigious Royal Society Newton Advanced Fellowship for her research in Special Functions and Orthogonal Polynomials.

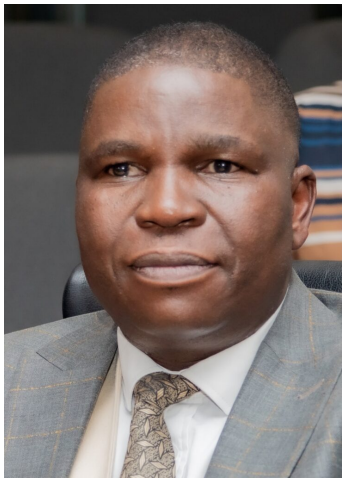
## Linking infectious diseases models with data in the presence of dynamic policies

FARAI NYABADZA*

*University of Johannesburg*
*fnyabadza@uj.ac.za*

SAMS Subject Classification Number: 10

The management of infectious diseases has evolved in the past few decades. It is now clear that policies can evolve as a disease progresses in a population, with a good example being the COVID-19 pandemic, where policies were dynamic and at times erratic. In this talk, we consider a number of mathematical models fitted to data and how the fitting is impacted by policy changes. We track the changes in policies using piece-wise systems of differential equations. The focus will be on how such models are formulated, analysed and interpreted. Thresholds that define disease persistence will be established and discussed. The implications of the policy changes are easily seen, and a discussion on how these changes impacted the epidemic is articulated. The results presented have crucial impact on how policy changes affected and continue to influence the trajectory of infectious diseases.



**Brief Bio:** Farai Nyabadza graduated with an undergraduate degree from the University of Zimbabwe majoring in Geology and Mathematics in 1993. He then did a Master of Science Degree in Mathematics from the University of Zimbabwe in 1998 and completed his Ph.D. in mathematical modelling of infectious diseases in 2003 from the University of Botswana. He proceeded to do a Post-Doc at the South African Center for Epidemiological Analysis (SACEMA), from where he joined Stellenbosch University. He is currently a essor of Applied Mathematics at the University of Johannesburg, and his research involves the applications of dynamical systems to infectious disease dynamics, substance abuse, and other biological systems. essor Nyabadza is the current Head of Department of the Department and Mathematics Applied Mathematics at UJ. He has successfully supervised 15 Ph.D. and 59 MSc students throughout his career. He has also written three academic books and has many collaborations globally. He is an editor of four Biomathematics journals and reviews a number of journal articles for national and international journals. He is the former President of the Southern Africa Mathematical Sciences Association, a regional body that thrives on developing Mathematical Sciences in the Southern African region. He is also passionate about motivational speaking and has written a book entitled titled, “Ordering your life for success: Lessons from the number line”.

## Senior Phase Mathematics: Do we mind the gap?

CRAIG POURNARA*

*University of the Witwatersrand*
*Craig.Pournara@wits.ac.za*

SAMS Classification: 16

It is well-known that learner performance in Senior Phase mathematics is poor. While more attention has been directed towards the Senior Phase in recent years, there is little convincing evidence of its impact at the level of the learner. What does it take to shift the needle in this phase? What changes in policy, in curriculum, in teaching and learning, and in assessment? In this talk I will argue that Senior Phase mathematics comes with challenges in all these areas. But, of equal importance, are the mathematical shifts that have to be navigated in the move to greater abstraction. Drawing from findings of several recent studies, I will challenge us to decide whether and how we mind this gap.

**Brief Bio:** Craig Pournara began his career as a high school teacher of Mathematics and Computer Science. He holds a PhD in Mathematics Education and has worked in pre-service and in-service maths teacher education for more than 25 years. He has served as Director of the Marang Centre for Maths and Science Education at Wits, and was a member of the Ministerial Task Team that developed the Teaching Mathematics for Understanding Framework. He is currently Director of the Wits Maths Connect Secondary Project where much of his work has focused on designing essional development and on researching the impact of that essional development on learners' attainment.
Craig is passionate about improving the teaching and learning of mathematics in South Africa. He longs for the day when more teachers teach mathematics with understanding, and more learners enjoy learning mathematics because it makes sense to them.

## Extensions of Ky Fan's best approximation theorem using some geometric properties of Banach spaces

MOOSA GABELEH *

*Ayatollah Boroujerdi University, Iran*
*gabeleh@abru.ac.ir*

SAMS Classification: 11

Let $X$ be a normed linear space and $Y$ be a nonempty subset of $X$. A point $x \in Y$ is said to be a fixed point for a mapping $T : Y \to X$ if $Tx = x$. In the case that $Y \cap T(Y) = \emptyset$, the fixed point equation $Tx = x$ does not possess a solution. In this case, it is interesting to ask whether there exists a point $p \in Y$ for which $p$ is closest to $Tp$. Such a point is said to be a best approximant point.

In 1969, Ky Fan presented some sufficient conditions to ensure the existence of a best approximant point. Fan's best approximation theorem states that if $Y$ is a nonempty, compact, and convex subset of a normed linear space $X$ and $T : Y \to X$ is a continuous mapping, then there is an element $p \in Y$ such that

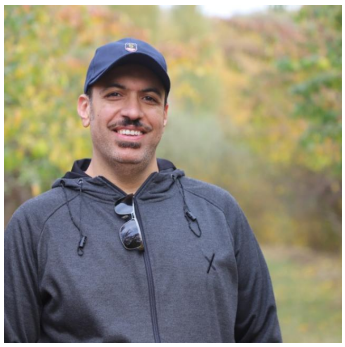
$$||p - Tp|| = \text{dist}(Tp, Y) := \inf\{||Tp - y|| : y \in Y\}.$$

Now assume that $Y$ and $Z$ are two nonempty subsets of a normed linear space $X$, and let $T : Y \to Z$ be a non-self mapping. In this situation, a point $p^* \in Y$ is called a best proximity point of the mapping $T$ provided that

$$||p^* - Tp^*|| = \text{dist}(Y, Z) := \inf\{||y - z|| : (y, z) \in Y \times Z\}.$$

It is worth noticing that if $Y \cap Z \neq \emptyset$, then the set of all best proximity points of $T$ coincides with the set of all fixed points of $T$. So, to obtain an extension of Fan's best approximation theorem, we need to assume that $Y \cap Z = \emptyset$.

In this talk, we will present some existence results of best proximity points for various classes of non-self mappings by using some appropriate geometric properties of Banach spaces, and then we apply such existence results to survey the existence of optimum solutions to systems of differential and integrodifferential equations.



**Brief Bio:** Moosa Gabeleh is an Iranian Mathematician, working in the field of Nonlinear Functional Analysis and more especially Best Proximity Point theory. He defended his PhD at Imam Khomeini International University (IKIU) in 2012 under the supervision of essor Ali Abkar on best proximity points for cyclic mappings. He did a post-doctoral internship at North-west University (NWU). He is currently a Full essor in the Department of Mathematics at Ayatollah Boroujerdi Unversity. He is the author of more than 100 scientific papers. In 2012 he was awarded the best paper in Iranian Functional Analysis Award from the Tusi Mathematical Research Group (TMRG).

## Models of Undergraduate Research in Mathematics

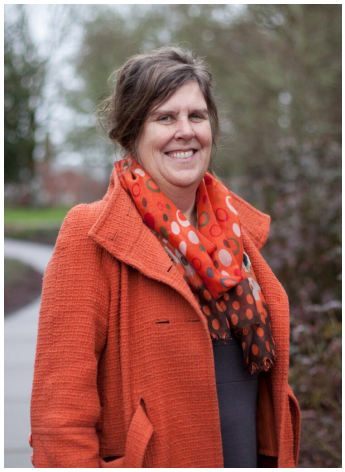
NANCY ANN NEUDAUER*

*Pacific University, USA*
*nancy@pacificu.edu*

SAMS Classification: 16

A well-mentored undergraduate research experience has been shown to be a high-impact practice, with particularly strong effects for students from minoritized groups. While common for decades amongst the lab sciences, undergraduate research has more recently been adopted and institutionalized in mathematics. I will present a few different models, and in particular, discuss the academic-year research and faculty essional development funded through the Center for Undergraduate Research in Mathematics (CURM). CURM has just been awarded its fourth NSF (National Science Foundation) grant, a $1.6 million five-year grant to support faculty and students with minigrants at universities and 2-year colleges throughout the United States.

CURM promotes academic-year undergraduate research in mathematics and statistics based upon a model consisting of (a) training faculty members to mentor students in research, (b) engaging students and a faculty mentor in research during the academic year at their own institution, (c) preparing students to succeed in graduate studies, and (d) advising faculty members on how to maintain consistent undergraduate research including finding resources for other funding sources.



**Brief Bio:** Nancy Ann Neudauer is the Thomas and Joyce Holce essor of Science and essor of Mathematics at Pacific University, Associate Secretary (responsible for the scientific program of national conferences) of the Mathematical Association of America (MAA), and Co-Director of the Center for Undergraduate Research in Mathematics (CURM). She received her MA and PhD in Mathematics, with a minor in Business and Law, and her BBA in Actuarial Science and Risk Management, all from the University of Wisconsin.
Nancy's research in matroid theory, graph theory, and combinatorics has been supported by grants from the Simons Foundation, the Fulbright Program, the National Science Foundation, and an endowed Research Chair, amongst others. She is Program Chair for the Cascadia Combinatorial Feast since 2001, was Visiting Mathematician to the national offices of the MAA, Director of the MAA Dolciani Mathematics Enrichment Grant Program for 13 years, a PI on the NSF-funded META Math (the Mathematical Education of Teachers as an Application of Mathematics) project, Associate Director for PNW Section NExT (New Experiences in Teaching) for 19 years, and recipient of a Distinguished Teaching Award and a Meritorious Service Award. As a Fulbright Specialist, her outreach extends to African Institute of Mathematical Sciences (AIMS) Centres in South Africa, Tanzania, Ghana, Cameroon, and Rwanda and she is the recipient of a Fulbright Global Scholars Award.

## Nullary monads, large zeros, and ideals

GEORGE JANELIDZE*

*University of Cape Town*
*george.janelidze@uct.ac.za*

SAMS Classification: 04

This talk is the third in the series started as [1] and [2], and partly summarized in [3]. We call the $0$ (=initial object) in a given category $\mathbf{C}$ *large* if pulling back along $0 \to 1$ determines a monadic functor. Surprisingly, when $\mathbf{C}$ is Bourn protomodular, the corresponding monad is what we call *essentially nullary*, which allows us to develop a theory of ideals in a non-pointed contexts. This is a wide generalization of the story of ideals in unital rings, where no proper ideal embedding is unital.

Note: (i) We will eventually require our ambient category $\mathbf{C}$ to be not just Bourn protomodular and having large zeros, but also to be Barr exact and finitely cocomplete. Such a category is semi-abelian in the sense of [4] if and only if it pointed, which was the reason for introducing the term "nearly semi-abelian" (used in [1] and [2]); however, since [3] I've changed it to "ideally exact", indicating "exact that admits a theory of ideals". (ii) Following [5] and related papers, we will briefly describe the connection with the theory of ideals in universal algebra, and especially the connection with what is more recently done in [6].

References:

[1] Janelidze, G (2023). Nearly semi-abelian categories, Invited talk at the online conference "Categories, Rings, and Modules" in Honour of Alberto Facchini, Padova, Italy.

[2] Janelidze, G (2023). A new direction in non-pointed categorical algebra, Invited talk at the online conference "Category Theory 2023", louvain-la-Neuve, Belgium.

[3] Janelidze, G (2023). Ideally exact categories, arXiv:2308.06574v2 [math.CT].

[4] Janelidze, G, Márki, L, and Tholen, W (2002). Semi-abelian categories, *J. Pure Appl. Algebra* **168**(2-3), 367–386.

[5] Janelidze, G, Márki, L, and Ursini, A (2007). Ideals and clots in universal algebra and in semi-abelian categories, *J. Algebra* **307**(1), 191–208.

[6] Ursini, A (2013). Normal subalgebras, I, *Appl. Categ. Structures* **21**(3), 209–236.

## Correspondence in Relevant (Modal) Logic

WILLEM CONRADIE*

*School of Mathematics, University of the Witwatersrand*
*National Institute for Theoretical and Computational Sciences (NITheCS)*
*willem.conradie@wits.ac.za*

VALENTIN GORANKO

*Department of Philosophy, Stockholm University, Sweden*
*valentin.goranko@philosophy.su.se*

PETER JIPSEN

*Department of Mathematics, Chapman University, USA*
*jipsen@chapman.edu*

SAMS Classification: 19

Relevant logic [4] was created in an attempt to address certain problematized features of classical material implication, including the fact that a false antecedent implies any consequent, even if the subject matter of the antecedent is not related to that of the consequent. Accordingly, relevant logic does not derive any implications where the antecedent and consequent are not relevant to each other, and where relevance is formally captured by the requirement that antecedent and consequent share at least one propositional variable. The relational semantics for relevant logic is given by Routley-Meyer models and frames where the implication is interpreted with a ternary relation and which stand in a discrete duality to relevant algebras [5]. Axioms added to the basic relevant logic define subclasses of Routely-Meyer frames. We present results on the correspondence theory for relevant logic which effectively identify large classes of relevant axioms which define elementary frame classes [1,2]. We define the classes of inductive and Sahlqvist relevant formulas and present an algorithm, PEARL, which computes first-order frame correspondents for all these formulas. PEARL has been implemented in Python and is accessible online. We discuss recent extensions of this work to modal versions of relevant logic [3], including an extended algorithm, MPEARL, and its implementation.

References:

[1] Conradie, W and Goranko, V (2021). Algorithmic correspondence for relevance logics I. The algorithm PEARL. In Ivo Düntsch and Edwin Mares, editors, *Alasdair Urquhart on Nonclassical and Algebraic Logic and Complexity of Proofs*, 163–209, Springer.

[2] Conradie, W, Goranko, V and Jipsen, P (2021). Algorithmic correspondence for relevance logics, bunched implication logics, and relation algebras via an implementation of the algorithm PEARL. In *Proceeding of RAMiCS 2021*, 126–143, Springer.

[3] Fuhrmann, A (1990). Models for relevant modal logics, *Studia Logica*, **49**, 501–514.

[4] Routley, R, Meyer, R, Plumwood, V and Brady, R (1982). *Relevant Logics and its Rivals (Volume I)*, Ridgeview, CA.

[5] Urquhart, A (1996). Duality for algebras of relevant logics, *Studia Logica*, **56**(1/2), 263–276.

**On cone metric spaces, uniformities and completions**

MOSHOKOA S*

*Tshwane University of Technology*
*moshokoasp@tut.ac.za*

APHANE M

*Sefako Makgatho Health Sciences University*
*maggie.aphane@smu.ac.za*

MSIPHA CN

*Tshwane University of Technology*
*msiphacn@tut.ac.za*

LEHLOHONOLO LP

*Tshwane University of Technology*
*lehlohononlolp@tut.ac.za*

SAMS Classification: 13

The talk continues the study of cone metric spaces and their completions. We will focus on the relationship between cone metric spaces and b metric spaces. We also present the completions of both spaces. Finally, the classical problem of extending maps to the completion in metric spaces will in the context of cone metric spaces be presented.

References:

[1] Abdeljawad, T. (2010). Completion of cone metric spaces. *Hacet. J. of Math. Stat.* **39** (1) 67-74.

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**Trailblazers: Ismail Mohamed's mathematical contributions**

EDER KIKIANTY

*University of Pretoria*
*eder.kikianty@up.ac.za*

LOYISO NONGXA*

*University of the Witwatersrand*
*loyiso.nongxa@wits.ac.za*

SAMS Classification: 15

Ismail Mohamed's major contributions, which were in collaboration with Hermann Heineken, was to provide a procedure for constructing groups with prescribed characteristics. In particular, they constructed examples of non-nilpotent groups in which every subgroup is subnormal and nilpotent. These have become known as the Heineken-Mohamed groups. This construction led to settling a few questions posed, in the 1940s, by Kurosh and Cernikov in their survey of various generalisations of nilpotency. He also studied properties of series of subgroups of a group $G$ that are constructed from arbitrary subgroups of automorphisms of a group.

## Singly Generated Quasivarieties and Residuated Structures

T. MORASCHINI

*University of Barcelona*
*tommaso.moraschini@ub.edu*

J.G. RAFTERY*

*University of Pretoria*
*james.raftery@up.ac.za*

J.J. WANNENBURG

*University of Pretoria*
*jamie.wannenburg@up.ac.za*

SAMS Classification: 35

A quasivariety K of algebras has the joint embedding property (JEP) iff it is generated by a single algebra $\mathbf{A}$. It is structurally complete iff the free $\aleph_0$-generated algebra in K can serve as $\mathbf{A}$. A consequence of this demand, called 'passive structural completeness' (PSC), is that the nontrivial members of K all satisfy the same existential positive sentences. We prove that if K is PSC then it still has the JEP, and if it has the JEP and its nontrivial members lack trivial subalgebras, then its relatively simple members all belong to the universal class generated by one of them. Under these conditions, if K is relatively semisimple then it is generated by one K-simple algebra. We also prove that a quasivariety of finite type, with a finite nontrivial member, is PSC iff its nontrivial members have a common retract. The theory is then applied to the variety of De Morgan monoids, where we isolate the sub(quasi)varieties that are PSC and those that have the JEP, while throwing fresh light on those that are structurally complete. The results illuminate the extension lattices of intuitionistic and relevance logics. For further details, see [1].

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## Elementary equivalence in positive logic via prime products

T. MORASCHINI

*Universitat de Barcelona*
*tommaso.moraschini@ub.edu*

J.J. WANNENBURG*

*University of Pretoria*
*jamie.wannenburg@up.ac.za*

K. YAMAMOTO

*Akademie věd České republiky*
*yamamoto@cs.cas.cz*

SAMS Classification: 19

Positive model theory focuses on formulas preserved by homomorphisms [3]. These are “positive existential formulas” constructed from atomic formulas and falsum, using only conjunction, disjunction, and the existential quantifier. The Keisler-Shelah Isomorphism Theorem states that structures are ”elementarily equivalent” if and only if they have isomorphic ultrapowers, originally proven under the Generalized Continuum Hypothesis (GCH) by Keisler [1].

We say two structures are “positively equivalent” if they satisfy the same positive existential sentences. We present a construction called a “prime product” that replaces the index set of an ultraproduct with a poset and uses a “prime filter” of the poset’s upward-closed sets. This generalization not only preserves positive existential formulas, but also the universal closure of implications between them, referred to as “basic h-inductive sentences.” Consequently, it becomes possible to characterize classes of models of h-inductive theories as those closed under isomorphisms, prime products, and ultraroots.

The central result of this talk establishes that, under the GCH, two structures satisfy the same positive existential sentences if and only if they have isomorphic prime powers of ultrapowers. This conclusion holds even in the absence of the GCH, provided that prime powers are replaced by prime products. However, ultrapowers remain essential, as structures can be positively equivalent without having isomorphic prime powers. These findings are detailed in [2].

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## A refined definable $(\mathrm{p}, \mathrm{q})$-theorem

GARETH BOXALL*

*Stellenbosch University*
*DSI-NRF Centre of Excellence in Mathematical and Statistical Sciences (CoE-MaSS), South Africa*
*gboxall@sun.ac.za*

CHARLOTTE KESTNER

*Imperial College London*
*c.kestner@imperial.ac.uk*

TSINJO RAKOTONARIVO

*Stellenbosch University*
*DSI-NRF Centre of Excellence in Mathematical and Statistical Sciences (CoE-MaSS), South Africa*
*tsinjo@sun.ac.za*

SAMS Classification: 19

The definable $(\mathrm{p}, \mathrm{q})$-conjecture for NIP theories is a statement in pure model theory with strong links to combinatorics. It led to several papers that established special cases of the conjecture and developed relevant techniques. Then, in 2022, Itay Kaplan gave a complete proof.

We give a refined version of this result which, in appropriate settings, allows us to replace the concept "not empty" with more refined notions such as "has dimension at least $\mathrm{d}$".

One of the big achievements of model theory in the last century was the study of what are called stable theories. A good example is the first-order theory of algebraically closed fields of some fixed characteristic. However, several important and well-behaved first-order theories are not stable. The class of NIP theories contains the class of stable theories but includes also the first-order theory of the real field $\mathbb{R}$ and the first-order theory of the $\mathrm{p}$-adic field $\mathbb{Q}_{\mathrm{p}}$, for each prime $\mathrm{p}$.

The definable $(\mathrm{p}, \mathrm{q})$-theorem for stable theories had been known for a long time. By proving it for NIP theories, Kaplan demonstrated that this aspect of the good behaviour of stable theories extends to the more general NIP setting. Our refinement adds to this picture.

## Nullital categories

ZURAB JANELIDZE*

*Stellenbosch University and NITheCS*
*zurab@sun.ac.za*

MICHAEL HOEFNAGEL

*Stellenbosch University and NITheCS*
*mhoefnagel@sun.ac.za*

SAMS Classification: 04, 35

In this talk we give an interpretation to an isomorphism

$$A/(B \times C) \approx (A/B)/C$$

in a pointed category with finite products. We show that it is implied by the isomorphism

$$(A \times B)/A \approx B,$$

which defines categories with normal projections [1]. We give a counterexample to show that the two properties are not equivalent to each other. Finally, we give various characterizations of the first property, including a characterisation of varieties of universal algebras satisfying it. One of the categorical characterisations reveals an intimate link with unital categories in the sense of [2], because of which we call pointed categories in which the former isomorphism holds, *nullital categories*.

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## On the Huq-commutativity of normal monomorphisms

JAMES RICHARD ANDREW GRAY*

*Stellenbosch University*
*jamesgray@sun.ac.za*

TAMAR JANELIDZE-GRAY

*University of Cape Town*
*tamar.janelidze-gray@uct.ac.za*

SAMS Classification: 04

It is well known and easy to prove that if $\mathrm{K}$ and $\mathrm{L}$ are normal subgroups of a group $\mathrm{G}$ and $\mathrm{K} \cap \mathrm{L} = 0$, then each element in $\mathrm{K}$ commutes with each element of $\mathrm{L}$. This fact has several known generalizations to categories.

An immediate generalization is obtained in the context where there is a suitable notion of a commutator $[-,-]$ defined for normal subobjects, (which is commutative and) satisfies the property that if $\mathrm{K}, \mathrm{L}$ are normal subobjects of an object $\mathrm{X}$, then $[\mathrm{K}, \mathrm{L}] \leq \mathrm{K}$. In this context, if $\mathrm{K}$ and $\mathrm{L}$ are normal subobjects of $\mathrm{X}$, then $[\mathrm{K}, \mathrm{L}] \leq \mathrm{K} \wedge \mathrm{L}$. Therefore if $\mathrm{K} \wedge \mathrm{L}$ is trivial it immediately follows that $[\mathrm{K}, \mathrm{L}]$ is trivial, which implies that $\mathrm{K}$ and $\mathrm{L}$ commute. This is the case for the Huq commutator in a normal ([5]) unital category ([1]). An alternative generalization was obtained by D. Bourn (Theorem 11 [3]) in the context of pointed protomodular category [2] (also introduced by D. Bourn): he proved that if $\mathrm{k}$ and $\mathrm{l}$ are Bourn-normal monomorphism with the same codomain and the meet of $\mathrm{k}$ and $\mathrm{l}$ is $0$, then $\mathrm{k}$ and $\mathrm{l}$ Huq-commute [4]. Recall that in a pointed finitely complete category, a Bourn-normal monomorphism is essentially the zero class of an internal equivalence relation.
The main aim of this talk is to show that there is a single categorical context, and single proof that implies these two different known facts. In fact we show that Bourn's result can be generalized to the wider context of a unital category satisfying, the condition which requires a morphism to be a monomorphism as soon as it's kernel is zero. This context is sufficiently wide so that it also includes every normal unital category which implies that the former result also becomes a special case. In doing so we produce an alternative criteria for when a pair of Bourn-normal monomorphisms commute in a unital category, which closely resembles Proposition 2.6.13 of [1].

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## On spectral spaces of ideals in algebras

AMARTYA GOSWAMI

*University of Johannesburg*
*agoswami@uj.ac.za*

SAMS Classification: 29, 13, 34

The purpose of this talk is to highlight specific aspects related to the spectral spaces of distinguished classes of ideals in various algebraic structures.

## Finite convex geometries via lattices and digraphs

ANDREW CRAIG*

*University of Johannesburg*
*acraig@uj.ac.za*

MIROSLAV HAVIAR

*Matej Bel University (Slovakia) & University of Johannesburg*
*miroslav.haviar@umb.sk*

KLARISE MARAIS

*University of Johannesburg & University of Amsterdam (The Netherlands)*
*klarise.marais@gmail.com*

SAMS Classification: 34

Previously, we provided a characterisation of a class of digraphs that provide dual representations of finite semidistributive lattices [2]. In this talk, we explore conditions on the TiRS digraphs dual to finite lattices that satisfy weakened versions of modularity. As a result, we are able to describe a new class of finite digraphs which represent finite convex geometries [3]. This description goes via the representation of finite convex geometries as finite lattices that are join-semidistributive and lower semimodular (cf. [1]).

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## A representation of odd Sugihara chains via weakening relations

ANDREW CRAIG

*University of Johannesburg*
*acraig@uj.ac.za*

CLAUDETTE ROBINSON*

*University of Johannesburg*
*claudetter@uj.ac.za*

SAMS Classification: 34

Sugihara monoids are commutative distributive idempotent residuated lattices with an order-reversing involution. Odd Sugihara monoids are those for which the monoid identity is fixed by the involution.

In this talk we will present a relational representation of odd Sugihara chains. The elements of our algebras are weakening relations on a poset which comes from a twisted product of a poset consisting of two densely embedded copies of the rationals with its dual. An order automorphism mapping an element in one of the densely embedded coppies of the rationals to the corresponding element in the other copy is the key to ensuring that the identity element of the monoid is fixed by the involution.

Our construction essentially combines the method of Maddux [3] with recent work [1]. There we were able to construct concrete distributive quasi relation algebras and give a definition of representable distributive quasi relation algebras. The elements of our algebras are weakening relations and the monoid operation is given by relational composition. Quasi relation algebras were introduced by Galatos and Jipsen [2] as a generalisation of relation algebras. One of the main reasons to try and generalise the class of relation algebras is the fact that the variety of relation algebras does not have a decidable equational theory.

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**Split extensions and semidirect products in algebraic categories**

EDWARD INYANGALA

*Sol Plaatje University*
*edward.inyangala@spu.ac.za*

SAMS Classification: 04

In the category of groups there is an equivalence between group actions and split extensions (which are split epimorphisms in this case). This equivalence is obtained via the semidirect product construction. The split epimorphism corresponding to an action $\mathrm{B} \longrightarrow \mathrm{Aut(X)}$ is the semidirect product projection $\mathrm{X} \rtimes \mathrm{B} \longrightarrow \mathrm{B}$.

In [1], a theory of split extensions of unitary magmas is developed. The theory includes defining such extensions and describing them via suitably defined semidirect product, yielding an equivalence of categories between the category of split extensions and the (suitably defined) category actions of unitary magmas on unitary magmas. The aim of this talk is to investigate analogous results in the context of monoids with operations [2], categories of interest in the sense of [3], groups with operations introduced by T. Porter [4] and more generally, non-semiabelian categories.

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## Tensor Pointfree Topology

CHARLES NSUKUKAZIFANI MSIPHA

*Tshwane University of Technology*
*msiphacn@tut.ac.za*

SAMS Classification: 04

A subobject $\mathrm{S}$ of the tensor unit $\mathrm{I}$ of a monoidal category $\mathbb{M}$ is said to be idempotent when $\mathrm{S} \otimes \mathrm{S} \cong \mathrm{S}$. In [1] and [2] the authors showed that the opens of a base space in categories for example, of Hilbert modules and of sheaves, can be recovered as idempotent subobjects of the tensor unit in any braided monoidal category. They revealed the spatial structure of morphisms into the tensor unit of a monoidal category. Their work resulted in another way of getting to pointfree topology not precursed by point set topology. Their results has generated some interests in investigating how topological notions like for example the compactness of frames (locales) can be defined in this setting.

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## Free objects in analytic categories

WALT VAN AMSTEL*

*University of Pretoria*
*sjvdwvanamstel@gmail.com*

MARCEL DE JEU

*Leiden University*
*mdejeu@math.leidenuniv.nl*

SAMS Classification: 04

The categorical notion of a free object is found in many areas of mathematics. Due to results from universal algebra, free objects are abundant in the context of algebra. Indeed, these results imply that free objects exist in every '*algebraic category*', where algebraic category is understood to mean a collection of algebraic objects of the same type along with the appropriate structure-preserving homomorphisms. This abundance of free objects stands in contrast with the situation in analysis, where free objects are known to not exist in many standard '*normed categories*' of interest to analysts. Despite this, there has recently been a surge of interest in *free Banach lattices*: It was shown in [3] that the free Banach lattice over a set exists. Other constructions, like the free Banach lattice over a Banach space or the free Banach lattice over a lattice have subsequently been studied, see [2], and [1]. In this talk, we discuss recent results relating to the existence of free objects in categories which are of interest in analysis. This includes a uniform approach for constructing free objects in normed categories. Beyond this, an inverse limit construction is used to produce free objects in '*locally convex categories*'. This talk is aimed at a category theory audience. However, it should be noted that the presenter is *not* a category theorist. It is the presenter's hope that the audience could alert the presenter to deeper results in category theory underpinning the work presented, if indeed there are deeper results at play.

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**Koszulness of Torelli Lie algebras**

JOSÉ SÃO JOÃO*

*Stockholm University and KTH Royal Institute of Technology*
*zealvro98@outlook.com*

SAMS Classification: 01, 02, 04

In early 2023, Kupers and Randal-Williams [1] published a paper answering a conjecture in Algebraic Topology and Algebraic Geometry proposed by Hain [2] (see also [3]). They proved that the Torelli Lie algebra, $\mathrm{Gr}^*_{\mathrm{lcs}}(\mathfrak{t}_{g,1})$, is Koszul in weight $\leq g/3$. In this talk, I will explain the statement of the result and give a broad overview of the proof. Moreover, it will be explained how we can generalise their arguments to give us a method of doing homotopy theory and commutative algebra in functor categories. This more generalised setting has applications to other structures in Algebraic Topology, for example in the homotopy theory of operads.

References:

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## On extensivity of morphisms in categories

EMMA THEART*

*Stellenbosch University*
*etheart@sun.ac.za*

SAMS Classification: 04

Extensivity, in the sense of [1], is a fundamental categorical concept which captures the behavior of coproducts and their relationship to pullbacks within the category of sets. Apart from the category of sets, examples of extensive categories include the categories of topological spaces, $G$-sets, partially ordered sets, and graphs. While extensivity is a property of an entire category, there are many examples of non-extensive categories which still exhibit extensive behavior within certain classes of morphisms. An example of this phenomenon can be found in the class of morphisms with trivial kernels in the category of pointed sets. Coextensive categories serves as the dual concept to extensive categories. Within this context, we can also narrow our focus to individual morphisms. Notably, in the category of groups, a non-coextensive category, the class of product projections of centerless groups exhibits coextensive behavior [2]. In this talk, our objective is to introduce the concept of an extensive morphism, as well as its dual counterpart, the coextensive morphism. We will explore these categorical concepts in various categories. Particularly, we will look at extensive morphisms in pointed categories, recognising that extensivity typically excludes pointedness. Additionally, we will delve into some general results associated with these concepts. The main result of this talk will be to show how coextensive categories can be characterised in terms of monomorphisms being coextensive, given suitable categorical conditions.

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## Grothendieck topologies and subforms

ROY FERGUSON *

*Stellenbosch University*
*royf326@gmail.com*

ZURAB JANELIDZE

*Stellenbosch University*
*zurab@sun.ac.za*

SAMS Classification: 04

A form, the theory of which is developed in [1], is a faithful amnestic functor. In this talk we develop a form-theoretic approach to Grothendieck topologies. For a category $\mathbb{C}$ the form of sieves, $\mathscr{S}$, has as fibres all sieves in $\mathbb{C}$. Any Grothendieck topology on $\mathbb{C}$ can be seen as a subform of $\mathscr{S}$. We formulate sieve-independent criteria characterising which subforms of $\mathscr{S}$ are Grothendieck topologies and explore this characterisation for superforms other than $\mathscr{S}$. A morphism $\mathrm{f}$ is an element of a sieve $\mathrm{S}$ if and only if $\mathrm{S}$ is above $\mathrm{f}$ in the order induced on fibres in $\mathscr{S}$. We refer to this notion as subordination. Subordination usually defines a morphism of forms from a form to the form of sieves. We characterise all forms for which subordination defines a Grothendieck topology.

One of the primary motivations for form theory is to provide a self-dual context to study nonabelian mathematical structures. We present a characterisation of all forms isomorphic to Grothendieck topologies which makes no reference to $\mathscr{S}$ or any superform. This allows us to define Grothendieck bitopologies: a Grothendieck toplogy which is isomoprhic to the dual of a Grothendieck topology. We characterise Grothendieck bitopologies as certain forms of $\mathscr{N}$-exact pairs [2] and offer some examples of these.

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## Császár Structures and Pre-Nearness in Frames

BAKULIKIRA CLAUDE IRAGI *

*University of the Western Cape*
*3418199@myuwc.ac.za*

DAVID HOLGATE

*University of the Western Cape*
*dholgate@uwc.a.za*

SAMS Classification: 13

In the realm of spaces, Á. Császár [3] developed the well-known theory of syntopogenous structure on a set. His aim was to establish a comprehensive framework that encompasses the study of topological, proximal, and uniform structures simultaneously.

In [2], Herrlich introduced the concept of "nearness" with the idea of unifying various notions of topological structures. He further suggested a correspondence between symmetric syntopogenous spaces and nearness spaces.

While all of these structures are well-defined in pointfree topology, to the best of our knowledge, there is no clear correspondence between nearness structures and Császár structures within the context of frames.

In this talk, we intend to bridge this gap. We will establish a correspondence between the category of pre-nearness frames [4] and a novel category of semi-Császár structures that we introduce. Furthermore, when considering the concept of quasi-uniformities in a frame, we demonstrate that interpolative Császár structures are in a one-to-one correspondence with the bases of a quasi-uniformity. Lastly, in accordance with [1] and [4], we establish a relationship between entourage quasi-uniformities and pre-uniformities within the framework of frames.

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**Formal Classification of Yoneda-Quillen Exact Categories**

KISHAN DAYARAM

*University of Johannesburg*
*220157896@student.uj.ac.za*

SAMS Classification: 04

Yoneda-Quillen exact categories provide a self-dual context in which the classical diagram lemmas of homological algebra may be established (see [1,2]). In [3], forms (which are faithful, amnestic functors) are used to define the noetherian form which is a self-dual context (different from that of Yoneda-Quillen exact categories) which covers all group-like structures. Furthermore, it was shown in [4] that the classical diagram lemmas of homological algebra may be established in the context of noetherian forms. In this talk, we will show that Yoneda-Quillen exact categories may be classified using forms. We will also show that Yoneda-Quillen exact categories may be generalized in a way that the classical diagram lemmas still hold.

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## Extensivity for sum structures

ROY FERGUSON *

*Stellenbosch University*
*royf326@gmail.com*

MICHAEL HOEFNAGEL

*Stellenbosch University*
*mhoefnagel@sun.ac.za*

ZURAB JANELIDZE

*Stellenbosch University*
*zurab@sun.ac.za*

EMMA THEART

*Stellenbosch University*
*21068038@sun.ac.za*

SAMS Classification: 04

Sum structures [3] are a class of monoidal structures which generalise the monoidal structure of coproduct. Here, existence is relaxed in the colimit condition. In this talk we will explore a generalisation of the theory of extensivity to the context of a category equipped with a sum structure. We refer to this notion as sum-extensivity and show that it has two equivalent formulations analogous to those of extensivity as set out in [1]. We demonstrate that a number of standard results for extensive categories generalise naturally to the sum-extensive context and we explore examples of categories which are sum extensive, but lack coproducts. In doing so, we demonstrate that sum-extensivity provides us with a means to apply the apparatus of extensivity to a broader context. We also detail examples of sum-extensive categories which are pointed and explore some properties in this setting. This is significant because only the trivial category is extensive and pointed. Finally, we explore restrictions of sum-extensive categories to certain classes of morphisms $\mathcal{M}$ and relate this to $\mathcal{M}$-extensivity as first described in [2].

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## Towards Free Localic Algebras

NATHAN TSHAKATUMBA*

*University of South Africa*
*46716866@mylife.unisa.ac.za*

PARTHA PRATIM GHOSH

*University of South Africa*
*ghoshpp@unisa.ac.za*

INDERASAN NAIDOO

*University of South Africa*
*naidoi@unisa.ac.za*

SAMS Classification: 04

The purpose of this talk is to show that the underlying object functor from the models of a Lawvere theory to the base category creates limits and coequalisers of all parallel pairs of homomorphisms whose underlying pairs admit a split coequaliser. Furthermore, we show that for a small complete category with a *well behaved* proper factorisation structure, the underlying functor admits a left adjoint and the category of such models is precisely monadic over the base category in the sense of Beck's theorem. In particular, this establishes the existence of free localic algebras for any Lawvere theory, generalising the known results for the existence of free localic groups.

## On variants of local connectedness in frames

MACK MATLABYANA*

*University of Limpopo*
*marck.matlabyana@ul.ac.za*

PINKIE SHIKWENI

*University of Limpopo*
*pinkie.shikweni@ul.ac.za*

HLENGANI SIWEYA

*University of Limpopo*
*hlengani.siweya@ul.ac.za*

SAMS Classification: 13

In this talk we give some generalisations of local connectedness in frames and present some characterisations. These generalisations are extensions of the generalisations of locally connected spaces in [1] and [2] to pointfree topology.

References:

[1] Kohli, JK, Singh, D, Tyagi, BK (2013). Between local connectedness and sum connectedness, *Comm. Math.*, **53**(1), 3–14.

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**Freely generated frames, compactness and the strong Hausdorff property for locales**

JOSHUA DAVID PASSMORE*

*University of the Western Cape*
*4259977@myuwc.ac.za*
*joshuapassmore@me.com*

SAMS Classification: 34

A meet-semilattice $\mathrm{A}$ can be extended to a frame by considering the lattice $\mathfrak{D}\mathrm{A} := \{\mathrm{X} \subseteq \mathrm{A} \mid \mathrm{X}$ is a down-set$\}$ of down-sets of $\mathrm{A}$. This process can be made functorial by sending each meet-semilattice homomorphism $\mathrm{f} : \mathrm{A} \to \mathrm{B}$ to a frame homomorphism $\mathfrak{D}\mathrm{f} : \mathfrak{D}\mathrm{A} \to \mathfrak{D}\mathrm{B}$, where $\mathfrak{D}\mathrm{f}(\mathrm{X}) := \downarrow \mathrm{f}[\mathrm{X}]$. It will be shown that $\mathfrak{D} : \mathbf{MSLat} \to \mathbf{Frm}$ is left adjoint to the forgetful functor $\mathrm{U} : \mathbf{Frm} \to \mathbf{MSLat}$ and thus $\mathfrak{D}$ is a free functor over $\mathbf{MSLat}$ (see [3]). Coverages on meet-semilattices will be discussed and $\mathrm{C}$-ideals will be introduced. The collection $\mathrm{C}\text{-Idl}(\mathrm{A})$ of all $\mathrm{C}$-ideals of a meet-semilattice $\mathrm{A}$ will be shown to be a sublocale of $\mathfrak{D}\mathrm{A}$. Following P.T. Johnstone in [2], it will be shown that $\mathrm{C}\text{-Idl}(\mathrm{A})$ and, in particular, $\mathfrak{D}\mathrm{A}$ are freely generated frames. As an application of freely generated frames, using $\mathrm{C}$-ideals, coproducts in $\mathbf{Frm}$ (equivalently, products in $\mathbf{Loc}$) will be constructed. The strong Hausdorff property for locales (as was introduced by J.R. Isbell in [1]) will be defined and, using part of the Kuratowski-Mrówka Theorem for locales (see [3]), we present and prove a result of J.J.C. Vermeulen (see [4]) which states that compact sublocales of strongly Hausdorff frames are closed (which is satisfactory since this mimics an analogous result in classical topology).

References:

[1] Isbell, J.R. (1972). Atomless parts of spaces, *Math. Scand.*, **31**, 5–32.

[2] Johnstone, P.T. (1982). *Stone spaces*, Cambridge Univ. Press, Cambridge.

[3] Picado, J. and Pultr, A. (2012). *Frames and locales: Topology without points*, Front. Math., vol. 28, Springer, Basel.

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## Compression with wildcards all k-models of a Binary Decision Diagram

M. WILD

*Stellenbosch University*
*mwild@sun.ac.za*

Y. SEMEGNI*

*North West University*
*Yves.Semegni@nwu.ac.za*

SAMS Classification: 06

Given the DNF form of a Boolean function $\phi$ on $n$ variables, e.g.,

$$\phi = \overline{x}_1 x_2 \overline{x}_3 \vee \overline{x}_1 \overline{x}_2 x_3 \vee \overline{x}_1 x_3 \vee x_2 x_3 x_4 \vee x_4 x_5 x_6 \vee \overline{x}_4 \overline{x}_5 \overline{x}_6 \vee x_1 x_3 \overline{x}_4 \vee x_3 x_5 \overline{x}_6,$$

deciding whether the equation $\phi = 1$ is consistent or not is known to be NP [1]. However given a Binary Decision Diagram [2] $B$ of the Boolean function $\phi$, all $N$ many $\phi$-models of Hamming-weight $k$ can be enumerated in polynomial time [3]. In this project, based on novel wildcards introduced in [4], we obtain a compressed enumeration of these models.

References:

[1] Crama, Y, and Hammer, P.L (2011). *Boolean Functions*, Enc. of Math. Appl. 142, Cambridge University Press.

[2] D.E. Knuth, The Art of Computer Programming (2012). *Combinatorial Algorithms, Volume 4A, Part 1*, Addison Wesley.

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## On $\mathrm{P}$ frames and their generalisations

TD NGOAKO*

*University of Limpopo*
*thabongoako1996@gmail.com*

MZ MATLABYANA

*University of Limpopo*
*marck.matlabyana@ul.ac.za*

HJ SIWEYA

*University of Limpopo*
*hlengani.siweya@ul.ac.za*

SAMS Classification: 13

In this talk, we give characterisations of $\mathrm{P}$-frames and some of their generalisations. In particular, we consider essential $\mathrm{P}$-frames and almost $\mathrm{P}$-frames, $\mathrm{F}$-frames and $\mathrm{P_F}$-frames. We extend the notion of $\mathrm{P_F}$-spaces as it appears in [1] and show that every cozero complemented $\mathrm{P_F}$-frame is basically disconnected. The class of $\mathrm{P_F}$-frames is contained in the class of $\mathrm{F}$-frames, and we show that $\mathrm{L}$ is a $\mathrm{P_F}$-frame if and only if it is an essential $\mathrm{P}$-frame which is also an $\mathrm{F}$-frame.

References:

[1] Azarpanah, F, Mohamadian, R and Monjezi, P (2021). On $\mathrm{P_F}$-spaces, *Topology Appl.*, **302**, 107821.

## Open maps over the Salbany compactification

MBEKEZELI NXUMALO

*Rhodes University*
*sibahlezwide@gmail.com*

SAMS Classification: 13

Recall from [1] that associated with a topological space $X$ is a space $\mathcal{U}X$ (whose points are ultrafilters on $X$) called the Salbany compactification of $X$ and the embedding $\eta_X : X \to \mathcal{U}X$, defined by $x \mapsto \{A \subseteq X : x \in A\}$, called the Salbany map of $X$. For every continuous function $f : X \to Y$, there is a continuous function $\mathcal{U}f : \mathcal{U}X \to \mathcal{U}Y$, called the Salbany lift of $f$, satisfying $(\mathcal{U}f) \circ \eta_X = \eta_Y \circ f$. If a continuous function $f : X \to Y$ has a stably compact codomain $Y$, then there is a Salbany extension $F : \mathcal{U}X \to Y$ of $f$, not necessarily unique, such that $F \circ \eta_X = f$.

In this talk, we discuss openness and nearly openness of (i) the Salbany map of a space, and (ii) the Salbany lift and Salbany extension of a continuous function.

References:

[1] Salbany S., Ultrafilter spaces and compactifications, Port. Math. **57**(4) (2000), 481–492.

**Characterizing some Generalizations of Lindelöf Frames**

SIYABONGA DUBAZANA*

*University of KwaZulu-Natal*
*siasydney@outlook.com*

SIMO MTHETHWA

*University of KwaZulu-Natal*
*MthethwaS4@ukzn.ac.za*

SAMS Classification: 13

In this presentation, I will introduce the study of regular frames with the property that whenever two closed sublocales with a compact intersection cover the whole frame, then at least one of the closed sublocales in the cover is Lindelöf. Frames with this characteristic will be called $\mathcal{L}\mathrm{J}$-frames [1]. One advantage of studying $\mathcal{L}\mathrm{J}$-frames is that it provides grounds to explore some interactions between connectedness and the Lindelöf property. The class of $\mathcal{L}\mathrm{J}$-frames contains all connected frames without points. We show that $\mathcal{L}\mathrm{J}$-frames are a generalization of Lindelöf frames. A typical spatial example of an $\mathcal{L}\mathrm{J}$-frame that is not Lindelöf is the frame of open sets of the long line considered with the topology generated by the lexicographical order. Sufficient conditions under which a frame is an $\mathcal{L}\mathrm{J}$-frame, and several characterizations of these frames are given. Among other things, we establish a property of completely regular $\mathcal{L}\mathrm{J}$-frames via remainders in the regular Lindelöfication.

References:

[1] Dubazana, S S and Mthethwa, S S (2023). Characterizing some Generalizations of Lindelöf Frames, *Rocky Mt. J. Math.*, Under Review.

## How to build uniform real-valued functions on a frame using scales

ANA BELÉN AVILEZ *

*University of the Western Cape*
*anabelenavilez@gmail.com*

SAMS Classification: 13

In classical topology one can build a real-valued function on a space using a family of open sets indexed on a dense set. This can be mimicked in point-free topology, and it is the easiest way to construct real-valued functions on a frame $\mathrm{L}$. One uses a family of elements (or sublocales) of $\mathrm{L}$, satisfying some conditions, to define a continuous real-valued function on the frame $\mathrm{L}$.

In this talk we will describe how to construct uniformly continuous real-valued function on a frame $\mathrm{L}$. To do so, we will introduce the notion of farness between elements (and sublocales) of a frame, and we will characterize uniform continuity in terms of the farness relation. This, in turn, will allow us to define a uniform scale, that is a scale that defines a uniformly continuous function.

This talk is based on the articles [1] and [3], and more details can be found in [2].

References:

[1] Arrieta, I. and Avilez, A. B. (2023). A general insertion theorem for uniform locales, *J. Pure Appl. Algebra*, **227**(7), 107320.

[2] Avilez, A. B. (2022). *A Point-Free Study of z-Embeddings, More General Classes of Localic Maps, and Uniform Continuity* [PhD thesis], University of Coimbra.

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## On the posets of connected elements in a locally connected frame

BERNARDUS A. WESSELS*

*Stellenbosch University*
*24199427@sun.ac.za*

JEAN F. DU PLESSIS

*Massachusetts Institute of Technology*
*jeandp@mit.edu*

ZURAB JANELIDZE

*Stellenbosch University*
*zurab@sun.ac.za*

SAMS Classification: 34

In pointfree topology, one abstracts the poset of open subsets of a topological space, by replacing it with a frame (a complete lattice, where finite meet distributes over arbitrary joins). In this talk, we propose an analogous abstraction of the poset of connected subsets of a topological space. To motivate this abstraction, we use it to characterise posets of connected elements in a locally connected frame, and establish the intuition underlying the proof of this result.

**Dually open morphisms and interior operators**

FIKREYOHANS SOLOMON ASSFAW *

*University of the Western Cape*
*fassfaw@uwc.ac.za*

SAMS Classification: 04

The formal theory of categorical interior operators was introduced by Vorster in [4]. These operators have played a vital role in the development of Categorical Topology. In particular, they have been used to study the notion of openness in an arbitrary category; see [1, 2].

Working in an arbitrary category endowed with a fixed $(\mathcal{E}, \mathcal{M})$-factorization system such that $\mathcal{M}$ is a fixed class of monomorphisms, we introduce and study dually open morphisms with respect to a given categorical interior operator $\mathrm{i}$. We discuss some basic properties of these morphisms. In particular, it is shown that the class of dually $\mathrm{i}$-open morphisms is stable under pullback along $\mathrm{i}$-open monomorphisms. We prove that dually $\mathrm{i}$-open morphisms are a generalization of both $\mathrm{i}$-codense morphisms (that are studied in [1]) and inversely open maps that are defined in [3]. We provide examples in topology and algebra.

References:

[1] Assfaw, F. S and Holgate, D (2021). Codenseness and openness with respect to an interior operator, *Appl. Categ. Structures*, **29**(2): 235-248.

[2] Castellini, G (2015). Interior operators, open morphisms and the preservation property, *Appl. Categ. Structures*, **23**(3): 311-322.

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## On seed number of R-subgroups of Beidleman near-vector spaces

P. DJAGBA*

*Nelson Mandela University*
*s228379946@mandela.ac.za*

A. L. PRINS

*Nelson Mandela University*
*abraham.prins@mandela.ac.za*

SAMS Classification: 29

Nearfields are skewfields (also called division rings) that lack the distributive law on one side. They were first studied by Dickson in 1905 and turned out to have applications to geometry and automata theory. For an overview of the subject, see book (1). More recently, contributions to the theory of Beidleman near-vector spaces were made in (2). In this work, we studied linear mappings of Beidleman near-vector spaces and explored their matrix representations using R-bases of R-subgroups. Additionally, we developed algorithms for determining the seed number and seed sets of R-subgroups within finite-dimensional Beidleman near-vector spaces.

References:

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[2] Djagba, P and Howell, K-H (2020). The subspace structure of finite dimensional Beidleman near-vector spaces, *Linear Multilinear Algebra*, **68**, 2316–2336.

**A survey of the results on derivations of evolution algebras**

JACQUES RABIE*

*University of Cape Town*
*jacques.rabie@uct.ac.za*

JUANA SANCHEZ-ORTEGA

*University of Cape Town*
*juana.sanchez-ortega@uct.ac.za*

YOLANDA CABRERA CASADO

*Universidad de Málaga*
*yolandacc@uma.es*

CÁNDIDO MARTÍN GONZÁLEZ

*Universidad de Málaga*
*candido_m@uma.es*

DOLORES MARTIN BARQUERO

*Universidad de Málaga*
*dmartin@uma.es*

SAMS Classification: 29

Evolution algebras are nonassociative, nonunital, but commutative algebras with a canonical basis such that the product of any two different canonical basis elements is 0. These algebras have many applications in various branches of mathematics, and derive their name from their ability to model asexual reproduction [1].

Recently, there has been a surge in interest in derivations on these structures (see for example [2-4]). This talk serves as a survey of these results, as well as an introductory glimpse into the study of ternary derivations of these structures.

References:

[1] Tian, JP (2007). *Evolution algebras and their applications*, Springer.

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## On the reverse to Cayle's representation theorem for monoids

GIDEO JOUBERT

*University of Stellenbosch*
*22930701@sun.ac.za*

SAMS Classification: 12,15

Cayley's representation theorem for a monoid $\mathrm{M}$ states that the monoid can be represented as a submonoid of the endofunction monoid of its underlying set $\mathrm{X}$ [1]. In this talk I will consider reverse question: given a submonoid $\mathrm{Y}$ of the monoid of endofunctions $\mathrm{End}(\mathrm{X})$ of some set $\mathrm{X}$, when is there a monoid structure on $\mathrm{X}$ so that $\mathrm{Y}$ is its representation? I will then make use of the fact that if we make a slight generalization to the reverse representation property, namely that $\mathrm{Y}$ is isomorphic to a submonoid of $\mathrm{End}(\mathrm{X})$, we can study this phenomenon from the viewpoint of monoid (or rather semigroup) actions. Finally, I will use the correspondence between semigroup actions and transformation semigroups [2] to re-contextualise my results and provide an answer to my initial question.

References:

[1] Jacobson, N (2009). *Basic algebra I*, 2nd Ed., Dover Publications.

[2] Arbib, M A, Krohn, K, and Rhodes, J L (1968). Algebraic theory of machines, languages, and semi-groups. Academic Press.

## Tableaux and Decision Procedures for Many-Valued Modal Logics

GUY AXELROD*

*University of the Witwatersrand*
*g.axelrod1@gmail.com*

WILLEM CONRADIE

*University of the Witwatersrand*
*National Institute for Theoretical and Computational Sciences (NITheCS)*
*willem.conradie@wits.ac.za*

SAMS Classification: 19

We present novel results expanding on the work done by Melvin Fitting in [1] and [2]. In [1], Fitting introduces a framework for many-valued modal logics where modal formulas are interpreted via generalized Kripke models in which both the propositional valuation and the accessibility relation take on values from some Heyting algebra of truth values. For a fixed arbitrary finite Heyting algebra, $\mathcal{H}$, [2] presents a signed semantic tableaux system that is sound and complete with respect to the $\mathcal{H}$-valued modal logic of all $\mathcal{H}$-frames. We go on to consider the many-valued generalizations of frame properties such as reflexivity, transitivity and seriality (as presented in [3]) and give parameterized tableaux systems that are sound and complete with respect to the $\mathcal{H}$-valued modal logics of the classes of $\mathcal{H}$-frames satisfying each of these properties. Further, we present a prefixed tableaux system allowing us to define an implementable decision procedure deciding the validity and satisfiability problems of the above mentioned many-valued modal logics.

References:

[1] Fitting, Melvin (1992). Many-Valued Modal Logics II. *Fund. Inform.*, **17**, 55–73.

[2] Fitting, Melvin (1995). Tableaus for Many-Valued Modal Logic. *Studia Logica*, **55**, 63–87.

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## Gluing index categories and generalized gluing data functors

DAMAS MGANI*

*University of Stellenbosch*
*25122738@sun.ac.za*

SOPHIE MARQUES

*University of Stellenbosch*
*smarques@sun.ac.za*

SAMS Classification: 04

In this presentation, we present a novel approach to the concept of mathematical gluing, offering a unified perspective that applies to a wide array of mathematical structures. These structures encompass topological spaces, presheaves, sheaves, ringed topological spaces, locally ringed topological spaces, and schemes. Our approach hinges on introducing two key notions: the 'gluing index categories' and the 'generalized gluing data functors.' These concepts provide a rigorous and categorical framework for characterizing the gluing process.

In this discussion, we primarily focus on the categorical gluing of topological spaces and presheaves (resp. sheaves), showcasing how the resulting glued-up topological spaces and presheaves (resp. sheaves) can be succinctly described as a limit of a gluing data functor. This perspective not only facilitates a more abstract understanding of gluing but also casts a fresh light on this mathematical concept.

**Near-linear algebra**

SOPHIE MARQUES

*Stellenbosch University*
*smarques@sun.ac.za*

DANIELLA MOORE*

*Stellenbosch University*
*21321264@sun.ac.za*

SAMS Classification: 29

In this talk, we will present some fundamental results for near-vector spaces toward extending classical linear algebra to near-linear algebra. We will discuss key results on the subspace span of a near-vector space, which leads to the proof that any near-vector space subspace is itself a near-vector space. We will also present how any quotient of a near-vector space by a subspace is a near-vector space and the First Isomorphism Theorem for near-vector spaces.

References:

[1] Marques, S and Moore, D (2022). *Near-linear algebra*, **Arxiv:2210.03601**.

## On the Structure of the Mislin Genus of a Pullback

THANDILE TONISI

*University of the Witwatersrand*
*1242739@students.wits.ac.za*

RUGARE KWASHIRA

*University of the Witwatersrand*
*rugare.kwashira@wits.ac.za*

JULES MBA

*University of Johannesburg*
*jmba@uj.ac.za*

SAMS Classification: 04

The notion of genus for finitely generated nilpotent groups was introduced by Mislin. Two finitely generated nilpotent groups $Q$ and $R$ belong to the same genus set $\mathscr{G}(Q)$ if and only if the two groups are non-isomorphic, but for each prime $p$, their $p$-localizations $Q_p$ and $R_p$ are isomorphic. Mislin and Hilton introduced the structure of a finite abelian group on the genus if the group $Q$ has a finite commutator subgroup. In this study, we consider the class of finitely generated infinite nilpotent groups with a finite commutator subgroup. We construct a pullback $H_t$ from the $l$-equivalences $H_i \to H$ and $H_j \to H$, $t \equiv (i+j) \bmod s$, where $s = |\mathscr{G}(H)|$, and compare its genus to that of $H$. Furthermore, we consider a pullback $L$ of a direct product $G \times K$ of groups in this class. Here we prove results on the group $L$ and prove that its genus is non-trivial.

## Irreducibility and Sobriety in the Lower Vietoris Topology

SHIMON CORCOS*

*University of KwaZulu-Natal*
*shimoncorcos@gmail.com*

SAMS Classification: 13

The *Vietoris topology* is a well-known topology that may be defined on the set of non-empty subsets of any topological space [1]. This topology may be defined in terms of the *upper Vietoris topology* and the *lower Vietoris topology* [2]. In my PhD thesis, I am investigating irreducibility and sobriety [3] in these topologies; which are essential in areas of topology where dealing with Hausdorff spaces is a rare occurrence such as Point-free Topology [4]. For a given space $\mathrm{X}$, it is well-known that the set of non-empty closed sets of $\mathrm{X}$ with the lower Vietoris topology (i.e. the *Hoare powerspace* [5]) is sober [6] and it was recently shown that the upper Vietoris topology on the set of non-empty compact and saturated subsets of $\mathrm{X}$ (i.e. the *Smyth powerspace* [5]) is sober [7]. Aside from results on the spectral and Noetherian properties in the Hoare powerspace [8],[9], there are currently no other significant results on irreducibility or sobriety in these topologies. There are no significant results on irreducibility and sobriety in the Vietoris topology. In this presentation, I will present results I have obtained for the lower Vietoris topology.

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[7] Heckmann, R and Keimel, K (2013). Quasicontinuous Domains and the Smyth Powerdomain, *Electron. Notes Theor. Comput. Sci.*, **298**, 215–232.

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## Expansive homeomorphisms on quasi-metric spaces

MCEDISI SPHIWE ZWENI*

*North-West University*
*sphiwe.zweni@nwu.ac.za*

OLIVIER OLELA OTAFUDU

*North-West University,*
*olivier.olelaotafudu@nwu.ac.za*

SAMS Classification: 33

The investigation of expansive homeomorphisms in metric spaces began with Utz in 1950 [1]. Then after, several authors [2-4] have extensively studied this concept for different motivations. Since the notion of expansive homeomorphism of join-compact quasi-metric space has not yet been studied, this motivated us to study the concept of an expansive homeomorphism of a quasi-metric space $(\mathrm{X}, \mathrm{q})$. We generalize some results from metric point of view to quasi-metric settings. For instance, we prove that if an expensive homeomorphism on a quasi-metric space that has canonical coordinates, then the canonical coordinates are hyperbolic. Moreover, we study expansive measures on the Borel structure generated by $\tau(\mathrm{q}) \cup \tau(\mathrm{q}^{\mathrm{t}})$ on quasi-metric space $(\mathrm{X}, \mathrm{q})$. The study of this notion is also motivated by the fact that any expansive homeomorphism on quasi-pseudometric space is again expansive homeomorphism on its induced pseudometric space but the converse is not true in general.

References:

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## Hydraulic fracturing with fluid leak-off: Mathematical models and solutions

M.W. NCHABELENG*

*University of Pretoria*
*mathibele.nchabeleng@up.ac.za*

A.G. FAREO

*University of the Witwatersrand*
*Adewunmi.Fareo@wits.ac.za*

SAMS Classification: 20

Hydraulic fracturing is a well stimulation technique used in order to improve the productivity of crude oil and natural gas from underground reservoirs. The technique involves injecting fluid at ultra-high pressure into the reservoir rock in order to propagate pre-existing fracture networks or create new fractures. A crucial goal in hydraulic fracturing is the use of the high pressure fluid to continually create and propagate new fractures in the rock formation, in order to continually liberate the crude oil and natural gas that may have been trapped. However, because of the permeable nature of the rock formations, fluid leak-off into the formations do occur, the rate of which increases as more and more fractures are created. The effect of fluid leak-off on fracture extension or propagation is significant and several research in this area has been done. In this talk, a mathematical model for the propagation of a pre-existing hydraulic fracture with fluid leak-off in a permeable rock will be presented. The empirical Darcy law is used to describe the fluid leak-off through the permeable fracture interface into the rock formation. The fluid flow in the fracture is laminar and the fracture is driven by a viscous incompressible Newtonian fluid. When lubrication theory is applied to the fracturing fluid flow in the hydraulic fracture, a system of integro-differential equations for the fracture half-width and the leak-off depth is obtained. Similarity and numerical solutions obtained for the integro-differential system are presented.

## On the existence of global weak solutions to the 3D electrically conductive Rosensweig system and their convergence towards quasi-equilibrium

A. NDONGMO NGANA*

*Department of Mathematics and Statistical Sciences, North-West University, South Africa*
*ndongmoaristide@yahoo.fr*

P. A. RAZAFIMANDIMBY

*School of Mathematical Sciences, Dublin City University, Collins Avenue, Dublin D09W6Y4, Ireland*
*paul.razafimandimby@up.ac.za*

SAMS Classification: 03

In this talk, we present an electrically conductive Rosensweig model for ferrofluids whose Bloch-Torrey regularisation was studied by Hamdache and Hamroun in [Appl. Math. Optim. 81, no. 2, 479–509 (2020)]. Our main focus is on demonstrating the existence of weak solutions to the non-regularized model, provided that the electric conductivity meets certain smallness criteria. Consequently, our findings not only resolve an unresolved issue left by Hamdache and Hamroun but also support the notion that ferrofluids naturally have low electrical conductivity. The proof of our result is complex, relying on techniques like the Helmohltz-Leray decomposition of magnetic fields and the use of renormalized solutions for magnetization. Additionally, we present a rigorous and comprehensive explanation of how the global weak solutions converge to a quasi-equilibrium state in the relaxation time limit regime $\tau$ approaches zero.

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## The Influence of the Co-Dynamics Ebola–COVID–19 in the Population

ARÉNE JAURÈS OUEMBA TASSÉ*

*University of the Witwatersrand*
*arsene.ouembatasse@wits.ac.za*

JEAN LUBUMA

*University of the Witwatersrand*
*jean.lubuma@wits.ac.za*

BERGE TSANOU

*University of Dschang, Cameroon*
*bergetsanou@gmail.com*

SAMS Classification: 05

Although no co-infection Ebola-COVID-19 have been reported so far, the Ebola outbreaks which occured during COVID-19 pandemic produced fewer cases than in the past, while all the efforts were devoted to fight against COVID-19 and the other diseases, including Ebola were neglected. In order to understand this unexpected fact, we propose a coupled Ebola-COVID-19 model. The Ebola-only and the COVID-19 only models are analysed. We prove for each of them that the basic reproduction number is a sharp threshold which ensures the elimination or the persistence of the disease. The basic reproduction of the coupled model $\mathcal{R}_0$ is equal to $\max(\mathcal{R}_0^{\mathrm{E}}, \mathcal{R}_0^{\mathrm{C}})$, with $\mathcal{R}_0^{\mathrm{E}}$ and $\mathcal{R}_0^{\mathrm{C}}$, the basic reproduction numbers of Ebola and COVID-19, respectively. In the case of cross immunity, we prove that the Ebola-COVID-19 free equilibrium is locally asymptotically stable when $\mathcal{R}_0 < 1$. When either $\mathcal{R}_0^{\mathrm{E}} > 1$ or $\mathcal{R}_0^{\mathrm{C}} > 1$, one of the disease persists while the other dies out, while when both $\mathcal{R}_0^{\mathrm{E}}$ and $\mathcal{R}_0^{\mathrm{C}}$ are greater than one, the two diseases persist and be endemic. We fit the model to the reported data for the Kivu-EVD outbreak in 2018-2020, since this outbreak overlapped before and during COVID-19, we split the data in two phases. Our results show that, the occurence of COVID-19 lead (a) to a reduction by more than 52% of contacts between individuals; (b) to the reduction by half of the immigration; and (c) to the quickly burying of the Ebola-deceased individuals. These factors reduced the level of EVD and sped up its elimination.

## Geometric Singular Perturbation Approach to Glass Network

NGWU*, BENITHO A.

*Federal University Oye Ekiti, Ekiti State, Nigeria*
*benitho.ngwu@fuoye.edu.ng*

MBAH, GODWIN C.E.

*University of Nigeria, Nsukka, Nigeria*
*godwin.mbah@unn.edu.ng*

ISIENYI, SUNDAY U.

*Akanu Ibiam Federal Polytechnic, Unwana, Afikpo, Ebonyi State*
*ottaman15@gmail.com*

SAMS Classification: 05, 12

Glass network is a class of gene regulatory network (GRN) used to model the interactions and control existing among molecules, proteins and other transcription factors in a biological system. The activities of the genes and their regulatory effects are controlled through their concentration levels defined by threshold functions. Qualitative and numerical studies of this network have, in general, revealed a lot of properties such as existence of periodic orbits, stable oscillation, chaos and steady states among others. This study applied the method of geometric singular perturbation to investigate the behaviour of the network within the threshold neighbourhood (known as singular states). The dynamics of the network is transformed into slow and fast systems at the singular states to enable the expected investigation with the use of GSP. Filippov's first order theory, Fenichel's persistence theorem among others were used to obtain the desired results.

## An ultraspherical spectral element method for solving partial differential equations

EMMA NEL*

*Stellenbosch University*
*21865663@sun.ac.za*

NICK HALE

*Stellenbosch University*
*nickhale@sun.ac.za*

SAMS Classification: 23

We investigate the ultraspherical spectral element method for solving second-order partial differential equations in two dimensions. Moreover, a novel coordinate transformation is introduced to broaden the scope of the method, making it applicable to rectangular domains with circular holes (square donuts). The presented method is an integration of two approaches, namely the ultraspherical spectral method and the hierarchical Poincaré–Steklov (HPS) scheme. The ultraspherical method is a Petrov–Galerkin scheme that presents operators in the form of sparse and almost-banded matrices, enabling both stability and computational efficiency. The HPS method is a recursive domain decomposition strategy that enables fast direct solves. It merges solution operators and Dirichlet-to-Neumann operators between subdomains, enforcing continuity of the solution and its derivative across domain boundaries. The fusion of these two methods results in an accurate discretisation with an explicit direct solve that can be applied to problems with smooth solutions. A major advantage is the reuse of precomputed solution operators facilitated by the HPS scheme, enhancing the efficiency of elliptic solves within implicit and semi-implicit time-steppers. An existing implementation of the method is established as a software system, ultraSEM, which employs the HPS method to solve on rectangular and polygonal domains. We extend this implementation to allow for solving on domains with circular cavities. This extension relies on a nonlinear coordinate mapping and proves to work effectively, achieving geometric convergence and near machine level precision accuracy. We demonstrate the application of the method and new domain on time-dependent fluid dynamics examples.

## Development of Hermite-Based Eight-Step Elliptic-Type PDEs Direct Integrator

E.O. ADEYEFA*

*FEDERAL UNIVERSITY OYE-EKITI*
*emmanuel.adeyefa@fuoye.edu.ng*

E.O. OMOLE

*FEDERAL UNIVERSITY OYE-EKITI*
*omolez247@gmail.com*

SAMS Classification: 23

A new eight-step elliptic partial differential equation solver (EEPDES) is presented in this work. The EEPDES has its nodes within the eight-step interval and targets elliptic partial differential equations on the two-dimensional domain. A collocation approach is adopted to develop this method, while a Hermite polynomial is employed as the basis function. The interpolation points are carefully selected at the two desired points, and at all the suitably preferred grid and off-grid points. By uniting the resulting equations and evaluating them after some simplification, the classical EEPDES is obtained. Investigating the numerical properties of the EEPDES, it was confirmed that the EEPDES is zero-stable and consistent. The accuracy and efficiency of the EEPDES were established by solving varying elliptic partial differential equations.

## Optimal Production and Regulation of Gold Mining: A Stochastic Differential Game Approach

MASIMBA A. MUTAKAYA

*University of Limpopo, Department of Mathematics and Applied Mathematics*
*masimba.aspinas@gmail.com*

FARAI J. MHLANGA*

*University of Limpopo, Department of Mathematics and Applied Mathematics*
*farai.mhlanga@ul.ac.za*

SAMS Classification: 25

We treat production and regulation of gold mining on a stock driven by a geometric Brownian motion, and a geometric Lévy process. The mining company seeks to maximise the revenues generated from its extracting activities while the government aims to maximise its share of profits by enacting regulatory policies. The problem is formulated as a non-cooperative stochastic differential game. A verification theorem is presented and proved under the assumption that Fleming-Bellman-Isaacs (FBI) equations have a $C^{1,2,1}$ solution. We derive closed-forms for the value functions and optimal strategies. The models are calibrated using South African mining data. Numerical results are presented to illustrate the findings of the study as well as the economic interpretation.

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## Unsteady MHD Carreau fluid flow over an inclined stretching/shrinking sheet with viscous dissipation in the presence of aligned magnetic field, effective Prandtl number and heat generation

GEORGE BUZUZI*

*Cape Peninsula University of Technology*
*buzuzig@cput.ac.za*

WILLIAM MANAMELA

*Cape Peninsula University of Technology*
*manamelawi@cput.ac.za*

SAMS Classification: 23

Numerical investigations on the impact of aligned magnetic field, effective Prandtl number and heat generation on Unsteady MHD Carreau fluid flow over an inclined stretching/shrinking sheet is discussed. By applying suitable similarity transformations, the system of non-linear partial differential equations are converted to a system of non-linear ordinary differential equations. Numerical solutions are obtained using MATLAB bvp4c technique. The influence of the various pertinent parameters on the velocity, temperature, skin friction and heat transfer rate are discussed with the help of graphs and tables. Numerical computations reveal that for a stretching sheet escalating values of the effective Prandtl number suppresses the velocity and temperature profiles. The role of the effective Prandtl number on fluid velocity and fluid temperature is reversed when the sheet shrinks.

## Vibration of thin Functionally Graded Planar Structures

GT SEDEBO*

*Tshwane University of Technology*
*SedeboGT@tut.ac.za*

MY SHATALOV

*Tshwane University of Technology*
*ShatalovM@tut.ac.za*

We consider a functionally graded plate (FGP) composed of an arbitrary number of concentric sections of various radii and thicknesses. This structure is supported at the centre by a rigid stem. The FGP is described in terms of the Novozhilov-Goldenveizer theory of thin isotropic linear shells. This system vibrates both in-plane as well as out-of-plane. Hamilton's variational principle is used to derive equations of motion and boundary-continuity conditions. A fixed boundary at the inner edge and a free boundary on the outer edge of the plate are considered. The in-plane and out-of-plane vibrations of the plates are studied in the frame of thin plate models and analytical solutions in terms of Bessel's functions and modified Bessel's functions for the system are determined. To verify our assumptions, we consider an example consisting of two concentric plates, where the inner component is made of an aluminium alloy and the outer component is made of titanium. The eigenvalues and the associated eigenfunctions of the plate are illustrated graphically. For this FGP both in-plane and out-of-plane vibrations are excited with different circumferential wave numbers, where an elliptical in-plane wave form is considered for in-plane vibrations and a three fold symmetry wave form is considered for out-of-plane vibrations. In order to consider an application of the planar structure as a three-dimensional inertial navigation vibratory gyroscope, it is necessary to tune the eigenvalues of both in-plane and out-of-plane vibrations. The desired tuning between the two modes is achieved by means of a variation of radius and/or thickness of the functionally graded plate.

## Epidemics of Corruption: A Fractional Calculus Approach to Uncover Complex Dynamics

HANGWELANI MAGAU*

*Central University Of Technology*
*hmagau@cut.ac.za*

SAMS Classification: 10

Corruption, akin to a rampant epidemic, infiltrates societies, eroding trust and destabilizing governance structures. This study employs the innovative approach of Fractional Calculus to dissect corruption's intricate dynamics. Through Fractional differential and Integral operators such as Riemann-Lioville, Caputo-Fabrizio and Atangana-Baleanu, corruption dynamics is modeled. Integrating memory and non-local interactions, this approach provides a nuanced understanding of corruption's evolution. Additionally, we develop a numerical scheme for precise approximation, allowing us to simulate various scenarios of corruption epidemics at different fractional orders. This concise exploration, bridging mathematics and social sciences, yields profound insights into corruption's multifaceted nature.

## Numerical Method of Lines on Vibrating High Voltage Overhead Transmission Lines

JOUBERT SV

*Tshwane University of Technology*
*joubertsv@tut.ac.za*

SHATALOV MY

*Tshwane University of Technology*
*shatalovm@tut.ac.za*

BIDIE JNM *

*Tshwane University of Technology*
*bidiejnm@tut.ac.za*

SAMS Classification: 23

Numerous authors have investigated how to estimate the accuracy of the numerical solutions of higher order differential equations or systems of differential equations, in particular, first order systems. The Numerical Method of Lines (MOL) has received attention recently, being extended from ordinary differential equations (ODE) to partial differential equations (PDE). MOL has been used in the analysis of transmission lines, and thus will be in use in this study on vibrating high voltage overhead transmission lines. MOL dicretizes a differential equation in one or more dimensions while using analytical solution in the remaining direction, reducing the PDE to a system of first ODEs of initial value type. In order to use MOL, it is necessary that the partial differential equation be well-posed as an initial value problem in at least one dimension. MOL on the equation of motion of transmission lines results in a system that has a higher order system of ODEs. This study will investigate the possibility of applying MOL to complex higher order mixed derivative partial differential equations. An analytical solution will be determined by changing variables of the equation of motion to cater for change in length and crossectional area of the transmission line, thereafter apply the Garlekin-Kantorovich method. A comparison between the analytical solution and MOL numerical approximation will be done, and conclusions drawn.

## A discretization of the singularly perturbed Burgers–Huxley equation using the NSFD methodology

ESHETU B. DERZIE

*Adama Science and Technology University, Adama, Ethiopia*

JUSTIN B. MUNYAKAZI*

*University of the Western Cape*

TEKLE GEMECHU

*Adama Science and Technology University, Adama, Ethiopia*

SAMS Classification: 23

The simultaneous presence of a singular perturbation parameter and the nonlinearity raise the challenge of finding a reliable and efficient numerical solution for the singularly perturbed Burgers-Huxley equation. We propose a nonstandard finite difference scheme which is developed in the following manner. The time variable is discretized using the backward Euler method. This gives rise to a system of nonlinear ordinary differential equations which are then dealt with using the concept of nonlocal approximation. Through a rigorous error analysis, the proposed scheme has been shown to be parameter-uniform convergent. Simulations conducted on two numerical examples confirm the theoretical result. A comparison with other methods in terms of accuracy and computational cost reveals the superiority of the proposed scheme.

## Hydrodynamic Stability Analysis for MHD Casson Fluid Flow Through a Restricted Channel

SO ADESANYA*

*Redeemer's University, Nigeria*
*adesanyas@run.edu.ng*

TA YASUF

*Adeleke University, Nigeria*
*tundeayusuf04@gmail.com*

AT ADEOSUN

*Federal College of Education, Nigeria*
*shiteq99@gmail.com*

L RUNDORA*

*University of Limpopo, RSA*
*lazarus.rundora@ul.ac.za*

SAMS Classification: 20

Flow instability is a major challenge experienced in medical, engineering and industrial settings globally. For instance, flow instability linked with irregular cardiac output of the heart leads to organ malfunctioning in the medical field, it also encourages mechanical vibrations in the case of fluctuating flow rate, and several other applications. In this study, linear stability analysis is conducted to monitor the behavior of a small disturbance that is imposed on hydromagnetic Casson fluid that flows steadily through a saturated porous medium. A new variant of the Orr-Sommerfield equation is obtained and solved numerically by using spectral point collocation weighted residual approach with eigenfunction expansion of the Chebyshev polynomial as the admissible trial function. Based on the QZ algorithm, numerical results are obtained for wave and Reynold's numbers, wave velocity as functions of Magnetic field intensity and porosity shape parameters. Results are validated against previously released data. The biophysics of the heart, particularly in cardiac rhythm analysis, as well as several other medicinal and technical applications, is among the areas where the current work has applicability.

## A Numerical Study for Bagley–Torvik Equation

M. KGAROSE*

*University of South Africa*
*kgarom@unisa.ac.za*

H. JAFARI

*University of South Africa*
*jafarh@unisa.ac.za*

SAMS Classification: 23

One of the well-known fractional order differential equations is the Bagley–Torvik equation (BTE). In 1983, the BTE was introduced by Bagley and Torvik as an application of fractional calculus for studying viscoelastically damped structures and their vital role in applied science and engineering problems, especially any linearly damped fractional differential equations or systems. A general form of BTE is given as follows:

$$\begin{aligned} &Au''(t) + B\, D^{\frac{3}{2}}u(t) + Cu(t) = f(t), \quad t > 0, \\ &u(0) = u_0, \quad u'(0) = u_1, \end{aligned}$$

where $A \neq 0$, $B$ and $C$ are real numbers, with $D^{\frac{3}{2}}$ a Caputo fractional derivative and $f(t)$ is known function.

The BTE has been studied by many researchers. In this work we use two numerical methods for solving the Bagley–Torvik equation. We use the operational matrices based on Hosoya polynomial and the Adomian decomposition method for obtaining the numerical results.

## Solving singular nonlinear hyperbolic PDEs using domain-decomposition spectral collocation approach

M. P. MKHATSHWA*

*University of South Africa patsonmkhatshwa@gmail.com*

SAMS Classification: 23

In this work, we explore the application of multi-domain based spectral method for solving singular nonlinear hyperbolic partial differential equations (PDEs) over large computational domains. In the solution algorithm, the nonlinear PDEs are first re-worked into a linearized form of an iterative scheme, using quasilinearization method. The space domain is divided into overlapping sub-intervals of equal length, whereas the time domain is decomposed into equal non- overlapping sub-intervals. Bivariate Lagrange interpolating polynomials constructed using Chebyshev-Gauss-Lobatto points, are used to approximate the solutions to nonlinear PDEs. In the time domain, numerical solutions are computed independently in each sub-interval, and the continuity condition is used to get initial conditions for the subsequent sub-intervals. On the other hand, PDEs are solved simultaneously across the overlapping sub-intervals in space. The efficacy, stability and accuracy of the method are demonstrated by presenting error analysis, condition numbers and computational time for the solution of some examples of singular nonlinear hyperbolic PDEs arising in fluid mechanics. The accuracy of the scheme is also validated by comparing approximate solutions with existing exact solutions. The adoption of domain-decomposition technique is effective in minimizing numerical challenges that are associated with large matrices and ill-conditioned nature of the resultant coefficient matrix. The obtained results confirm that the method is highly accurate, stable, computationally cheaper, and converges rapidly when solving singular nonlinear hyperbolic PDEs using fewer grid points.

## New sets of soliton solutions for the generalized Whitham–Broer–Kaup–Boussinesq–Kupershmidt system

SHINA DANIEL OLONIIJU*

*Rhodes University*
*s.oloniiju@ru.ac.za*

SAMS Classification: 20

Nonlinear evolution equations play a crucial role in modeling various physical phenomena, such as fluid flow dynamics and plasma waves. Among them, the generalized Whitham-Broer-Kaup-Boussinesq-Kupershmidt (WBKBK) system, along with its special cases, has garnered significant attention in the study of soliton theory due to its intricate mathematical structure and broad range of applications. This study explores the properties and behavior of soliton solutions in the WBKBK system. The study presents six new classes of wavelike solutions of the WBKBK system and investigate the unique characteristics exhibited by dark and anti-dark solitons, as well as the phenomenon of forward and backward propagating solitons. These findings contribute to a deeper understanding of the fundamental dynamics of the WBKBK system and offer valuable insights into soliton solutions of the system.

**The analytical solutions of squeezing flow and heat transfer in a horizontal filtering channel.**

THABO J. ITUMELENG*

*North-West University*
*masegomafora07@gmail.com*

GABRIEL MAGALAKWE

*North-West University*
*Gabriel.Magalakwe@nwu.ac.za*

MODISAWATSONA LUCAS LEKOKO

*North-West University*
*Lucas.Lekoko@nwu.ac.za*

LETLHOGONOLO DADDY MOLELEKI

*North-West University*
*Letlhogonolo.Moleleki@nwu.ac.za*

SAMS Classification: 20

The dynamics of flow and heat transfer inside different industrial chambers play a vital role in understanding momentum and heat transfer during industrial operations. Analysis of such flow and heat transfer helps operators to understand how the system works. To better understand such flow and heat transfer inside the filter channel, the current study analytically investigates fluid flow and heat transfer through porous asymmetric filter channel under the influence of a uniform magnetic field while removing unwanted contaminations from the fluid (filtration process). The PDE model representing the internal flow and heat during filtration is transformed into a system of ODEs without changing the dynamics under investigation. The transformed equations are thereafter solved analytically using the perturbation approximation technique. The obtained solutions are used to find an optimal combination of dimensionless parameters which arise from flow and heat dynamics that leads to optimum outflow. Graphically analysis of flow and heat transfer dynamics are used to indicate an optimal parameters combination based on scientific evidence.

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## Analysis of a generalized cholera model and application of Sobolev spaces to the well-posedness of a reaction-diffusion model for cholera

VERONIQUE PHOLA MASUNDA*

*University of Pretoria*
*veroniquephola@gmail.com*

JEAN M-S LUBUMA

*University of the Witwatersrand*
*jean.lubuma@wits.ac.za*

SAMS Classification: 10

Cholera is an acute diarrhoeal infection caused by ingestion of food or water contaminated with the bacillus vibrio cholerae. This disease is of major public health importance and poses threat to the world, specifically in developing countries including South Africa where there is an ongoing outbreak since February 2023. In this talk, we consider a generalized model for the direct (human-to-human) and indirect (environment-to-human) transmissions of the cholera disease. We study the well-posedness of the model in the biologically feasible region. The qualitative analysis of the model starts by the computation of the basic reproduction number, $\mathcal{R}_0$, by using the next generation matrix. The main results read as follows: the unique disease-free equilibrium is globally asymptotically stable if $\mathcal{R}_0 < 1$ and unstable when $\mathcal{R}_0 > 1$. In the latter case, there exists a unique endemic equilibrium which is locally asymptotically stable. The proof of the global asymptotic stability of the endemic equilibrium is challenging. In the particular case of a linear incidence function and concentration of the pathogen, the endemic equilibrium is shown to be globally asymptotically stable by Lyapunov function techniques and LaSalle's invariance principle. Moreover, we use the Sobolev embedding theorem to prove the global existence of the solution of a related reaction-diffusion model.

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## Comparative Analysis of Deep Garlekin Method and Finite Difference Method for Solving PDEs in Portfolio Optimization

WILLIAM MALOSE MANAMELA

*Cape Penisula University of Technology*
*manamelawi@cput.ac.za*

SAMS Classification: 25

This study aims to investigate the performance of the deep learning algorithm: Deep Galerkin method (DGM), in comparison to the finite difference method (FDM). At first, we present the Merton problem framework and then derive the nonlinear partial differential equation and the associated optimal controls utilizing the Hamiltonian-Jacobian-Bellman equation. Furthermore, we solve the resulting PDES and optimal controls by implementing python code for both the DGM and FDM. The results demonstrates that in general the former outperform the later. We also observed that at various time points, DGM consistently provided more accurate results compared to FDM.

## An Agent-Based Model for Studying the Spread of COVID-19 in Mozambique: Pandemic Planning Implications of Population Mobility Patterns

ALFREDO ZACARIAS MUXLHANGA

*Eduardo Mondlane University, Maputo, Mozambique*
*alfredo.muxlhanga@uem.ac.mz*

SANSÃO AGOSTINHO PEDRO*

*Eduardo Mondlane University, Maputo, Mozambique*
*sansao.pedro@uem.ac.mz*

DAISUKE TAKAHASHI

*A.I.Systems Research Co., Kyoto, Japan*
*dtakahashi42@gmail.com*

The COVID-19 pandemic has become a new global public health crisis, and to large extent, its capacity to cross natural geographic barriers is attributed to human mobility and contact patterns which vary with time and specific locations. Therefore, an agent-based model (ABM) which relates populations mobility patterns in different locations in compliance with on site COVID-19 control measures is proposed to investigate how opening and closing protocols would have been best implemented in Mozambique.

For spatial dynamics, a survey was carried out in the city of Maputo as a case study to estimate populations mobility patterns and contact matrices among individuals in different locations (home, school, work place, worship place, market and any other place of gathering) during specific periods of the day (morning, afternoon and night) for both week days and weekends. Individuals are explicitly represented by agents associated to disease characteristics and their decision to remain or move to a new place is based on a probability estimated from the survey and on site declared control measures.

The results show that at $50\%$ of social distancing compliance, complete lockdown of schools, workplaces, worship places with exception of markets is the only scenario that result in the reduction and shift of the peak by $3\%$ and 3 days respectively. School closure showed significant effect that at $75\%$ and $85\%$ of social distancing adherence resulted in the reduction and shift of the peak by $15\%$ and 4 days, and $51\%$ and 24 days respectively. While closure of worship places rendered little effect due to limited frequency and duration of activities in a given location.

This study has demonstrated the use of simulation models to investigate the implementation of opening and closing policies for the control of COVID-19 pandemic at local scale by leveraging between the mobility of individuals and adherence to social distancing.

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## Supercharacter Theory of Metacyclic Groups

HADISEH SAYDI *

*North West University*
*54116236@mynwu.ac.za*

SAMS Classification: 15, 12

Supercharacter theory is developed by P. Diaconis and I. M. Isaacs as a natural generalization of the classical ordinary character theory [3]. Supercharacter theory of many finite groups such as the cyclic groups, the Frobenius groups, etc. are well studied and well-known [1]. In this paper we find some of supercharacter theories of metacyclic groups. The metacyclic group is an extension of a cyclic group by a cyclic group. The dihedral group, the dicyclic group and $\mathrm{U}_{6\mathrm{n}}$ are the metacyclic groups that we study [2,4,5,6].

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## Codegrees and element orders of finite groups

SESUAI Y. MADANHA *

*University of Pretoria*
*sesuai.madanha@up.ac.za*

SAMS Classification: 15

G. Qian [3] proposed a conjecture which states that if an element of a finite group $G$ has order $a$, then there exists an irreducible character of codegree $b$ of $G$ such that $a$ divides $b$. He showed that the conjecture holds for solvable groups. In [2], we settled the conjecture for almost simple groups. We shall discuss the proof. We will also discuss some recent work of Akhlaghi, Pacifici and Sanus in [1] where they proved that the conjecture holds for finite groups with a trivial Fitting subgroup.

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## Designs From The Maximal Subgroups And Conjugacy Classes Of $\mathrm{PSp}_4(q)$

CLARENCE K MOKALAPA*

*University of Limpopo*
*clarence.mokalapa@ul.ac.za*

AMIN SAEIDI

*University of Limpopo*
*saeidi.amin@ul.ac.za*

THEKISO T SERETLO

*North-West University*
*thekiso.seretlo@nwu.ac.za*

SAMS Classification: 15

In this talk, we construct some designs invariant under the simple groups $\mathrm{PSp}_4(q)$ for some prime power $q$. These designs are obtained from two maximal subgroups of indices $\frac{q^2(q^2+1)}{2}$ and $\frac{q^2(q^2-1)}{2}$, respectively and conjugacy classes of elements of $\mathrm{PSp}_4(q)$. The results are an application of what are known as method 1 and method 2 proposed by Key and Moori in $[1, 2]$ for the construction of designs invariant under finite simple groups.

References:

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## Constructing designs from fixed points of alternating groups

KEKANA MJ*

*University of Limpopo*
*Jankekana50@gmail.com*

SAEIDI A

*University of Limpopo*
*amin.saeidi@ul.ac.za*

SERETLO TT

*University of North West*
*thekiso.seretlo@nwu.ac.za*

SAMS Classification: 15

In this talk we construct some $1$–designs from Key-Moori Method 2 and J Moori Method 3. In Key-Moori Method 2, we have a technique from which a large number of non-symmetric $1$–designs could be constructed on a maximal subgroups and conjugacy classes of elements of finite groups. J Moori and A Saeidi studied the alternting groups with maximal subgroups of types $\mathrm{S}_{\mathrm{n}-1}$ . In this talk we will be looking at other maximal subgroups (particularly the maximal subgroup $\mathrm{A}_{\mathrm{n}-1}$ of $\mathrm{A}_{\mathrm{n}}$) which are not covered by J. Moori and A. Saeidi. Therefore, this work serves as an extension of the research conducted by J. Moori and A. Saeidi.

## Linear codes and designs from the unitary group $\mathrm{U}(3,3)$

TG NYIKADZINO*

*University of Limpopo*
*gtnyikadzino@gmail.com*

A SAEIDI

*University of Limpopo*
*amin.saeidi@ul.ac.za*

TT SERETLO

*North-West University*
*thekiso.seretlo@nwu.ac.za*

SAMS Classification: 15

In this presentation, we outline a method for computing linear codes invariant under primitive permutation groups. We will demonstrate that when the group $\mathrm{G}$ possesses a trivial Schur multiplier, every binary linear code that admits $\mathrm{G}$ as a permutation group can be regarded as a submodule of the permutation module within the primitive action of $\mathrm{G}$. As an illustrative example, we select the finite simple group $\mathrm{G} = \mathrm{U}(3,3)$ and identify the complete set of linear codes derived from its 2-representations. In the paper [1], the authors used the same approach to find all binary codes of the Mathieu group $\mathrm{M}_{11}$. It is worth noting that the collection of binary linear codes for $\mathrm{G} = \mathrm{U}(3,3)$ is already documented in [2].

In addition, we use the supports of the codes to construct certain designs that remain invariant under the action of $\mathrm{U}(3,3)$ and establish connections between these designs and the corresponding linear codes. Our computations are based on MAGMA [3].

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## Concatenated $\mathcal{T}$-*Direct* Codes for Multi-User $\mathbb{F}$-Adder Channel

MEENAKSHI DEVI*

*Sol Plaatje University*
*meenakshi.devi@spu.ac.za*

R. S. RAJA DURAI

*Jaypee University of Information Technology, India*
*rsraja.durai@juitsolan.in*

ASHWINI KUMAR

*Chitkara University, India*
*ashwinipatiyal@gmail.com*

SAMS Classification: 01

Let $\mathbb{F}_q$ denote the finite field with $q$ elements. While an LCD code $\mathcal{C} \subseteq \mathbb{F}_q^n$ satisfies the duality criteria $\mathcal{C} \cap \mathcal{C}^\perp = \{\mathbf{0}\}$ [1] with $\mathcal{C}^\perp$ being the dual code with respect to $\mathbb{F}_q^n$, each constituent code $\mathcal{C}_i$ of a $q$-ary $\mathcal{T}$-*Direct* code [2] is an LCD code in the sense that $\mathcal{C}_i^\perp$ is dual to $\mathcal{C}_i$ with respect to $\Lambda = \mathcal{C}_1 \oplus \mathcal{C}_2 \oplus \cdots \oplus \mathcal{C}_\mathcal{T} \subseteq \mathbb{F}_q^n$, $\mathcal{T} \leq n$. In this paper, a concatenation procedure for the class of $q^n$-ary $\mathcal{T}$-*Direct* codes is presented to obtain codes of larger lengths. Further, as the constituent codes of $2^n$-ary 2-*Direct* codes are known to provide an optimal coding solution for the two-user binary adder channel, the constructed concatenated $\mathcal{T}$-*Direct* codes are employed over the $\mathcal{T}$-user $\mathbb{F}_{q^n}$-adder channel to harness the usefulness of $\mathcal{T}$-*Direct* codes in multi-user coding. The coding and decoding procedures for the constructed class of concatenated codes over the *noiseless* as well as *noisy* $\mathcal{T}$-user $\mathbb{F}_{q^n}$-adder channel also presented. The fact that $\mathcal{C}_1 \oplus \mathcal{C}_2 \oplus \cdots \oplus \mathcal{C}_\mathcal{T} \subseteq \mathbb{F}_{q^n}^n$ allows and facilitates the required error correction for $\mathcal{T}$-*Direct* codes. With suitable parameters of concatenated $\mathcal{T}$-*Direct* codes $(\mathcal{C}_1, \mathcal{C}_2, \ldots, \mathcal{C}_\mathcal{T})$, the concatenation and coding procedures are illustrated through examples.

References:

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## On the deleted permutation modules of the alternating group $A_n$

AYOUB BASHEER*

*School of Mathematical and Computer Sciences, University of Limpopo (Turfloop), P Bag X1106, Sovenga 0727, South Africa*
*ayoubbasheer@gmail.com*

JAMSHID MOORI

*School of Mathematical Sciences, North-West University (Mafikeng), P Bag X2046, Mmabatho 2735, South Africa*
*jamshid.moori@nwu.ac.za*

SAMS Classification: 15

For every prime $p$ dividing the order of the alternating group $A_n$, there is an $A_n$-module $\mathbb{V}$ that we refer to it as the *fully deleted permutation module*. This module has dimension $n-1$ if $n$ is odd and $n-2$ if $n$ is even. A split extension group of the form $\mathbb{V}{:}A_n$ exists. In this talk we focus on the case when $n$ is odd and $p = 2$, that is the group $2^{n-1}{:}A_n := \overline{G}_n$. We completely determine the inertia factor groups and give some results on the character degrees of this group.

References:

[1] A. B. M. Basheer and J. Moori, *On the deleted permutation module of the alternating group* $A_n$, to be submitted.

## On the symplectic transvections of the symplectic group $\mathrm{Sp}(2n, F)$

S MKIVA*

*North-West University*
*soga.mkiva@nwu.ac.za*

J MOORI

*North-West University*
*jamshid.moori@nwu.ac.za*

SAMS Classification: 15

In [1] the presenter computes and constructs the Clifford-Fischer matrices of the affine subgroups of the classical linear groups. Classical linear groups comprise the general linear groups, symplectic groups, orthogonal groups and unitary groups. Some of the subgroups considered in [1] are affine subgroups of symplectic groups of the form: $2^9{:}\mathrm{Sp}(8,2) \leq \mathrm{Sp}(10,2)$, $2^6{:}\mathrm{Sp}(2,4) \leq \mathrm{Sp}(4,4)$ and $2^{10}{:}\mathrm{Sp}(4,4) \leq \mathrm{Sp}(6,4)$. Detailed general discussion on symplectic groups, including symplectic forms and symplectic spaces, can be found in [2], [3] and [5], among others. Since the symplectic group is generated by symplectic transvections, we study these transvections in great detail and prove a few results on them. Background theory on symplectic transvections is found in [4] and [6], among others. We present here some of our contributions on the study of symplectic transvections. In this note we study the symplectic transvections of the symplectic group $\mathrm{Sp}(2n, F)$, $n \in \mathbb{N}$ and $F = GF(q)$. We prove that if $\mathrm{Char}(F) = p$, $p$ a prime, then the order of a transvection in $\mathrm{Sp}(2n, F)$ is $p$. We further show that if $|F| = 2^k$, $k \in \mathbb{N}$, then there is only one conjugacy class of transvections in the group $\mathrm{Sp}(2n, 2^k)$.

References:

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## Three Methods of Generations of the Simple Group $\mathrm{PSL}(3,5)$

THEKISO SERETLO *

*North West University*
*thekiso.seretlo@nwu.ac.za*

SAMS Classification: 15, 12

A group is said to be $(\mathrm{l},\mathrm{m},\mathrm{n})$-generated, if it is a quotient group of the triangle group

$$\mathrm{T}(\mathrm{l},\mathrm{m},\mathrm{n}) = \langle \mathrm{x},\mathrm{y},\mathrm{z} \mid \mathrm{x}^{\mathrm{l}} = \mathrm{y}^{\mathrm{m}} = \mathrm{z}^{\mathrm{n}}, \mathrm{xyz} = 1 \rangle.$$

For a non-trivial conjugacy class of $\mathrm{G}$, $\mathrm{nX}$, we define, rank of $\mathrm{nX}$ in $\mathrm{G}$, denoted by $\mathrm{rank(G{:}nX)}$ to be the minimum number of elemenmts in $\mathrm{nX}$ that generate $\mathrm{G}$.
$\mathrm{G}$ is said to be $\mathrm{nX}$-complementary generated if for every $\mathrm{x} \in \mathrm{G}$ there exists $\mathrm{y} \in \mathrm{nX}$ such that $\mathrm{G} = \langle \mathrm{x},\mathrm{y} \rangle$.
In this talk we look at the results of the three methods of generation for the simple group $\mathrm{PSL}(3,5)$. This work is based on [1, 2, 3] with the foundation from [5, 6]. The notation used is from [4] and computations are done with [7].

References:

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## On the $(p, q, r)$-generations of the group $G_2(3)$

SEHOANA M.G*

*University of Limpopo*
*mahlare.sehoana@ul.ac.za*

SERETLO T.T

*North-west University*
*thekiso.seretlo@nwu.ac.za*

BASHEER A.B.M

*University of Limpopo*
*ayoubbasheer@gmail.com*

MOTALANE J.M

*University of Limpopo*
*john.motalane@ul.ac.za*

SAMS Classification: 15

A finite group $G$ is called $(l, m, n)$-*generated*, if it is a quotient group of the triangle group $T(l, m, n) = \left\langle x, y, z | x^l = y^m = z^n = xyz = 1 \right\rangle$. In [3], Moori posed the question of finding all the $(p, q, r)$ triples, where $p$, $q$ and $r$ are prime numbers, such that a non-abelian finite simple group $G$ is a $(p, q, r)$-generated. In this presentation we establish all the $(p, q, r)$-generations of the group $G_2(3)$. Since the order of the group $G_2(3)$ is $4245696 = 2^6.3^6.7.13$, the triples $(p, q, r)$ with the condition that $p \leq q \leq r$ are constructed from the set $\{2, 3, 7, 13\}$. Exceptions are triples $(2, 2, r)$ and $(2, 3, 7)$ which yield a non-simple group and a Hurwitz group [2], respectively. The Groups, Algorithms and Programming, GAP [1] and the Atlas of finite group representations [4] are used in our computations.

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## Constructing Fischer matrices of extensions $p^{1+2n}.G$ from their factor groups $p^{2n}.G$

ABRAHAM LOVE PRINS*

*Nelson Mandela University*
*abraham.prins@mandela.ac.za*

SAMS Classification: 28

Let $\overline{G} = p^{1+2n}.G$ be a finite extension of an extra-special $p$-group $P = p^{1+2n}$ by a group $G$. Since the center $Z(P)$ is characteristic in $P$ and hence normal in $\overline{G}$, the factor group $\overline{F} = \frac{\overline{G}}{Z(P)} \cong p^{2n}.G$ can be constructed, where $P_1 = p^{2n}$ is an elementary abelian $p$-group. In this talk, a method, called the *lifting of Fischer matrices*, is discussed to show how the Fischer matrices $M(g)$ [1] of $\overline{G}$ are constructed from the corresponding Fischer matrices $\widehat{M(g)}$ of $\overline{F}$. As an example, the ordinary character table of a subgroup $2_+^{1+8}{:}(U_4(2){:}2)$ [2] of the Conway simple group $Co_2$ is computed using the *lifting of Fischer matrices* method.

References:

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## On the Wiener Index of Orientations of Graphs

PETER DANKELMANN*

*University of Johannesburg*
*pdankelmann@uj.ac.za*

SAMS Classification: 06

The Wiener index $W(D)$ of a strong digraph $D$ with vertex set $V$ is defined by

$$W(D) = \sum_{(a,b)\in V\times V} d_D(a,b),$$

where $d_D(a,b)$ denotes the usual shortest path distance from $a$ to $b$ in $D$. This definition has been extended to the case that $D$ is not strong by defining $d_D(a,b)$ as $0$ if there is no path from $a$ to $b$ in $D$ (see (1), (2)).

In our talk we consider the Wiener index of (not necessarily strong) orientations of graphs. Knor, Škrekowski and Tepeh (1) conjectured that every tree has an orientation of maximum Wiener index in which there is one vertex $v$ with the property that for every vertex $w$ there is either a path from $w$ to $v$ or a path from $v$ to $w$. We disprove the conjecture.

We also settle a question (see (2)) on the computational complexity of finding an orientation of maximum Wiener index of a given input graph by showing that the corresponding decision problem is NP-complete.

The problem of finding an orientation of minimum Wiener index of a given graph is discussed briefly.

References:

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## The oriented diameter of graphs with given connected domination number and distance domination number

P. DANKELMANN

*University of Johannesburg*
*pdankelmann@uj.ac.za*

M.J. MORGAN

*University of KwaZulu-Natal*
*morganj@ukzn.ac.za*

E.J. RIVETT-CARNAC*

*University of Johannesburg*
*217093333@student.uj.ac.za*

SAMS Classification: 06

Let $G$ be a connected graph. The concept of the oriented diameter of a graph $G$ involves assigning a direction to each edge of $G$. One can think of it as "turning every street into a one-way street". The largest of the distances between two vertices in such an orientation is called its diameter. The smallest diameter of an orientation of a graph is its oriented diameter. It is computationally difficult (NP-complete) to find the oriented diameter of a given graph. Hence bounds on it are of interest.

We give bounds on the oriented diameter in terms of domination-type parameters of $G$, including the connected domination number and the $d$-distance connected domination number of $G$, which is a generalisation of the connected domination number.

In this talk, we show that every bridgeless graph with connected domination number $\gamma_c(G)$ has an oriented diameter of at most $2\gamma_c(G) + 3$ if $\gamma_c(G)$ is even and $2\gamma_c(G) + 2$ if $\gamma_c(G)$ is odd. Furthermore, we show that this bound is sharp and for every value of $\gamma_c(G)$ we construct a graph that attains this bound. We also present a bound on the oriented diameter in terms of the $d$-distance connected domination number of $G$. For constant $d$ our above bound is optimal apart from a factor of about $2$.

## Nearest Integer Continued Fractions: A road map to rational complex approximations in hyperbolic space

CARMINDA MARGARETHA MENNEN*

*University of the Witwatersrand*
*carminda.mennen@wits.ac.za*

SAMS Classification: 22

In this talk, we begin by introducing the nearest integer continued fraction with complex partial quotients. Instead of finding rational complex approximations for a complex irrational by truncating its nearest integer continued fraction, we use geometry in three-dimensional hyperbolic space to illustrate the process underlying the generation of these approximations.

References:

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## On Y-coordinates of Pell Equations which are Fibonacci Numbers

FAITH SHADOW ZOTTOR*

*University of Johannesburg*
*fszottor@uj.ac.za, fzottor@gmail.com*

FLORIAN LUCA

*University of the Witwatersrand*
*florian.luca@wits.ac.za*

SAMS Classification: 22

Let $d \geq 2$ be an integer which is not a square. We show that if $(F_n)_{n\geq 0}$ is the Fibonacci sequence and $(X_m, Y_m)_{m\geq 1}$ is the $m$th solution of the Pell equation $X^2 - dY^2 = \pm 1$, then the equation $Y_m = F_n$ has at most two positive integer solutions $(m, n)$ except for $d = 2$ when it has three solutions;

$$(m, n) = (1, 2), (2, 3), (3, 5).$$

## A note on a unified generalisation of Touchard and Fubini polynomials

SITHEMBELE NKONKOBE

*Sol Plaatje University*
*snkonkobe@gmail.com*

SAMS Classification: 06

Inserting bars inbetween blocks of an ordered set partition induces a barred preferential arrangement. In this study we introduce a unified generalisation of both Touchard and Fubini polynomials. We also propose some combinatorial identities. Furthermore we also give some combinatorial proofs using the notion of barred preferential arrangements. We also give some asymptotic results.

References:

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## Possible values of $d_0(G)$

FARAJI TIRAGA*

*Stellenbosch University*
*23638375@sun.ac.za*

RIANA ROUX

*Stellenbosch University*
*rianaroux@sun.ac.za*

SAMS Classification: 06

A set $S \subseteq V(G)$ is a dominating set of graph $G$ if every vertex in $V(G)\backslash S$ is adjacent to a vertex in $S$. The $k$-dominating graph $D_k(G)$ is a graph whose vertices are the dominating sets of $G$ of cardinality of at most $k$. Two dominating sets are adjacent in $D_k(G)$ if one can be obtained from the other by adding or deleting one vertex. For a graph $G$, we denote $d_0(G)$ as the minimum integer $k$ such that $D_k(G)$ is connected for all $k \geq d_0(G)$.

In this talk we give an overview of existing results on the possible values of $d_0(G)$ and discuss some open problems regarding well-covered and claw-free graphs.

## Binary trees with many total dominating sets are not unique

OPEYEMI OYEWUMI*

*Stellenbosch University*
*opeyemioluwaoyewumi@gmail.com*

RIANA ROUX

*Stellenbosch University*
*rianaroux@sun.ac.za*

STEPHAN WAGNER

*Uppsala University*
*stephan.wagner@math.uu.se*

SAMS Classification: 06

A subset $D$ of the vertex set $V(G)$ is a *total dominating* set of a graph $G$ if every vertex of $G$ has a neighbour in $D$. We show that in the family of binary trees (trees such that every vertex is either of degree $1$ or $3$), the binary caterpillar has the fewest number of total dominating sets. Interestingly, there are many binary trees with maximum number of total dominating sets, specifically when the number of vertices $n \equiv 0 \mod 4$.

## Optimal linear-Vizing relationships for (total) domination in graphs

MICHAEL HENNING*

*University of Johannesburg*
*mahenning@uj.ac.za*

PAUL HORN

*University of Denver*
*Paul.Horn@du.edu*

SAMS Classification: 06

In this talk, we discuss the following two problems, where $\gamma(G)$ and $\gamma_t(G)$ denote the domination and total domination numbers, respectively, of a graph $G$.

**Problem 1:** For each $\Delta \geq 3$, find the smallest value, $c_\Delta$, such that for every connected graph $G$ of order $n$, size $m$, domination number $\gamma(G) = \gamma$, and bounded maximum degree $\Delta(G) \leq \Delta$,

$$m \leq \left(\frac{\Delta + c_\Delta}{2}\right) n - \left(\frac{\Delta + c_\Delta + 2}{2}\right) \gamma.$$

**Problem 2:** For each $\Delta \geq 3$, find the smallest value, $r_\Delta$, such that for every connected graph $G$ of order $n \geq 3$, size $m$, total domination number $\gamma_t$, and bounded maximum degree $\Delta(G) \leq \Delta$,

$$m \leq \frac{1}{2}(\Delta + r_\Delta)(n - \gamma_t).$$

For all $\Delta \geq 3$, Rautenbach [1] in 1999 showed that $c_\Delta \leq \Delta$. Yeo [2] in 2007 showed that $0.05 \ln(\Delta) < c_\Delta$ and $0.1 \ln(\Delta) < r_\Delta \leq 2\sqrt{\Delta}$ for all $\Delta \geq 3$, and posed as an open problem to determine "whether $r_\Delta$ grows proportionally with $\ln(\Delta)$ or $\sqrt{\Delta}$ or some completely different function." In this talk, we determine the growth of $r_\Delta$, and show that both $c_\Delta$ and $r_\Delta$ are asymptotically $\ln(\Delta)$.

References:

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## Hosoya polynomial of the subdivided join

T.P. NCAMBALALA*

*University of the Witwatersrand*
*Thokozani.Ncambalala@wits.ac.za*

E.G. MPHAKO - BANDA

*University of the Witwatersrand*
*Eunice.Mphako-Banda@wits.ac.za*

SAMS Classification: 06

The Hosoya polynomials of diameter 1 and diameter 2 graphs are known. We extend the concept of a vertex join of a graph to a subdivided join. We then give the formula of the Hosoya polynomial of a subdivided join of a complete graph and the formula of the Hosoya polynomial of a subdivided join of diameter 2 graphs.

References:

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## The Number of $1$-Nearly Independent Subsets

ERIC O. D. ANDRIANTIANA

*Rhodes University*
*e.andriantiana@ru.ac.za*

ZEKHAYA B. SHOZI*

*Sol Plaatje University*
*zekhaya.shozi@spu.ac.za*

SAMS Classification: 06

Let $\mathrm{G}$ be a graph with vertex set $\mathrm{V(G)}$ and edge set $\mathrm{E(G)}$. A subset $\mathrm{I}$ of $\mathrm{V(G)}$ is an *independent vertex subset* if no two vertices in $\mathrm{I}$ are adjacent in $\mathrm{G}$. The number, $\sigma_0$, of vertex independent subsets of a graph is a popular graph invariant with rich literature. See the survey in [1], where it is called Merrifield-Simmons Index. In this paper we introduce a new direction of research that generalizes the notion of the number of vertex independent subsets. In particular, we wish to understand the behavior of the number, $\sigma_1(\mathrm{G})$ , of all subsets of the vertices of $\mathrm{G}$ that contain exactly one edge. We call those subsets 1-nearly independent vertex subsets. We prove a tight lower (resp. upper) bound on $\sigma_1$ for a connected (resp. general) graph. The extremal graphs achieving equality on each of these bounds are characterised. We also establish the explicit formulas for $\sigma_1$ of some special classes of graphs.

References:

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[2] E. Mphako-Banda. An introduction to the $\mathrm{k}$-defect polynomials. Quaestiones Mathematicae, 42(2):207–216, 2019.

## On members of Lucas sequences which are products of Catalan numbers

MARK SIAS*

*University of Johannesburg*
*msias@uj.ac.za*

SHANTA LAISHRAM

*Indian Statistical Institute*
*shanta@isid.ac.in*

FLORIAN LUCA

*University of the Witwatersrand*
*florian.luca@wits.ac.za*

SAMS Classification: 22

We show that if $\{U_n\}_{n\geq 0}$ is a Lucas sequence, then the largest $n$ such that $|U_n| = C_{m_1} C_{m_2} \cdots C_{m_k}$ with $1 \leq m_1 \leq m_2 \leq \cdots \leq m_k$, where $C_m$ is the $m$th Catalan number satisfies $n < 6500$. In case the roots of the Lucas sequence are real, we have $n \in \{1, 2, 3, 4, 6, 8, 12\}$. As a consequence, we show that if $\{X_n\}_{n\geq 1}$ is the sequence of the $X$ coordinates of a Pell equation $X^2 - dY^2 = \pm 1$ with a nonsquare integer $d > 1$, then $X_n = C_m$ implies $n = 1$.

References:

[1] Laishram, S, Luca, F, and Sias, M (2021). On members of Lucas sequences which are products of factorials, *Int.J.Number Theory*,**17**(06).

## Exploring Cyclotomic Field Extensions

ELIZABETH MREMA*

*Stellenbosch University*
*25138413@sun.ac.za*

SOPHIE MARQUES

*Stellenbosch University*
*smarques@sun.ac.za*

SAMS Classification: 22

Cyclotomic field extensions arise from adjoining a primitive $n^{th}$ root of the polynomial $x^n - 1, n \in \mathbb{N}$ to a given field $F$. Here, "primitive root" means an element with order $n$ in the group of solutions to $x^n - 1$. These extensions hold a central role in number theory and various mathematical disciplines, serving as a foundation for diverse applications. In number theory, cyclotomic extensions are instrumental in investigating Fermat's Last Theorem and the profound Kronecker-Weber Theorem, which provide profound insight into Abelian extensions. They also play a key role in exploring elliptic curves and modular forms. In more general, cyclotomic extensions serve as fundamental building blocks for constructing more complex field extensions. By understanding their structure, we might gain insights into these complex fields.

While extensive research has been done into cyclotomic extensions over the field of rational number $\mathbb{Q}$, comparatively limited attention has been directed toward fields of characteristic zero in general and positive characteristics.

In this talk, we thoroughly explore cyclotomic extensions of degrees of the form $2^m$ where $m \in \mathbb{N}$ over an arbitrary field. Our investigation encompasses various properties of these extensions, including the analysis of Galois structure, subfields, degrees, and the determination of minimal polynomials.

Moreover, we will highlight the connection we've established between certain coefficients of the minimal polynomial of a subfield within a cyclotomic extension $F(\zeta_{2^e}/F), e \in \mathbb{N}$ and well-known combinatorial numbers, namely Catalan numbers

## Split sequences of trees

ERIC O.D ANDRIANTIANA *

*Rhodes University*
*e.andriantiana@ru.ac.za*

VALISOA RAZANAJATOVO MISANANTENAINA

*University of Pretoria*
*valisoa.rakotonarivo@up.ac.za*

STEPHAN WAGNER

*Uppsala University, Sweden*
*stephan.wagner@math.uu.se*

SAMS Classification: 06

In this talk, we introduce the notion of split sequence Sp($\mathrm{T}$) of a tree. It is the non-increasing sequence of the order of the smallest connected component of $\mathrm{T} - \mathrm{e}$, as $\mathrm{e}$ varies among all possible edges in $\mathrm{T}$. It can be viewed as another form of the edge division vector in [On indices of Wiener and anti-Wiener type. Discrete Applied Mathematics, 251:290-298, 2018]. We will discuss characterisations of the split sequences and rediscover old results on distance based graph invariants by using split sequences. New results will also be mentioned.

## Sizes of flats of cycle matroids of complete graphs

C. KRIEL*

*University of the Witwatersrand*
*christo.kriel@wits.ac.za*

E. MPHAKO-BANDA

*University of the Witwatersrand*
*Eunice.mphako-banda@wits.ac.za*

SAMS Classification: 06

We show that the problem of counting the number of flats of size $k$ for a cycle matroid of a complete graph is equivalent to the problem of counting the number of partitions of an integer $k$ into triangular numbers. In addition, we give some values of $k$ such that there is no flat of size $k$ in a cycle matroid of a complete graph of order $n$. Finally, we give a minimum bound for the number of values, $k$, for which there are no flats of size $k$ in the given cycle matroid.

References:

[1] Kriel, C and Mphako-Banda, E (2022). Sizes of flats of cycle matroids of complete graphs, *Rev. Colombiana Mat.* , **56**(1), 63–75.

## Zero forcing in graphs

BRYAN CURTIS

*Iowa State University*
*bcurtis1@iastate.edu*

LESLIE HOGBEN

*Iowa State University*
*hogben@aimath.org*

RIANA ROUX*

*Stellenbosch University*
*rianaroux@gmail.com*

SAMS Classification: 06

Zero forcing is a propagation process on a graph. The propagation process may be describe by the repeated application of the following *colour change rule*: starting with an initial set of blue vertices, a blue vertex $v$ can change the colour of a neighbouring white vertex $w$ to blue if $w$ is the only white neighbour of $v$. A *zero forcing set* of $G$ is a subset $S$ of vertices such that if $S$ is the initial set of blue vertices the whole graph will eventually be coloured blue. The *zero forcing number* of a graph $G$, $Z(G)$, is the minimum cardinality of a zero forcing set.

We introduce the idea of $Z$*-irredundance*, which determines when a zero forcing set is minimal. In this talk we will discuss the relationships between zero forcing and $Z$-irredundance showcasing similarities and significant differences.

## On Robin's inequality

FLORIAN LUCA*

*Wits, STIAS*
*florian.luca@wits.ac.za*

PATRICK SOLÉ

*CNRS, Aix-Marseille University*
*patrick.sole@telecom-paris.fr*

SAMS Classification: 22

Robin conjectured that $\sigma(n) < e^{\gamma} n \log\log n$ holds for all $n > 7!$, where $\sigma(n)$ is the sum of the divisors of $n$ and $\gamma$ denotes the Euler constant. Robin showed that the validity of this inequality is equivalent to the Riemann Hypothesis. Here we show that the set of $n$'s failing this inequality has a very small counting function: the number of such $n \leq x$ is $O(x^{\varepsilon})$ for any $\varepsilon > 0$ and $x > x(\varepsilon)$.

## Exploring the causes of dyscalculia in foundation phase: a case study of a primary school in Pretoria

DINEO CHARMAINE MOLISE *

*University of Johannesburg*
*dineomolise95@gmail.com*

Purpose: In South Africa mathematics is not taught and learned effectively. Mathematical content can be difficult to understand for some children. This study identifies the underlying causes of poor academic performance among mathematics learners.

Design: Through classroom observations and interviews, this study utilized a qualitative approach to collect data. The interviews were conducted with teachers, educational psychologists, and learners in focus groups. The factors related to learners' difficulty grasping mathematical content were examined through virtual meetings with doctors and educational psychologists. A theme analysis was performed on the data to identify recurring themes.

Findings: An examination of dyscalculia, a condition that impairs the ability of learners to learn, is presented in this study. Commonly recognized aspects of dyscalculia are the difficulty in acquiring and remembering mathematics knowledge, and the difficulty in executing mathematical methods. These findings present some factors contributing to the difficulty of learning mathematical content. The neurodivergent of learners, poor visuospatial working memory, lack of concentration, lack of prior knowledge, environmental factors, math anxiety, and lack of early diagnosis of learning disabilities, were the major factors in the learning and teaching of mathematics. Research implication: The acquaintance of these factors responsible for arithmetic acquisition will raise awareness and provision to tackle the difficulties of arithmetic. The future trend will be to explore the neurodevelopment of learners to see if visuospatial working memory may account for the recognized link between frequent processing and mathematical performance in children with arithmetic difficulties.

## Advantages and Disadvantages of Technological Advances in Mathematics Education

FULUFHELO RATSHISINDI *

*Incandescent Group PTY (LTD)*
*f.ratshisindi@gmail.com*

SAMS Classification: 16

The relationship between technological advances and mathematics education has surely been a positive one in terms of simplifying and improving the speed of delivery of the content. However, attention is not given to what is lost when the teaching of mathematics is reliant on technological advances compared to the traditional teaching methods. This paper aims to outline disadvantages of technological advances in the teaching of mathematics. We also provide counter methods to minimise (even eliminate) the negative impact of technological advances on the teaching of mathematics and the development of mathematics as a discipline. We also give a summary of advantages of the technological advances and how they can be implemented in the teaching of mathematics in such a way that nothing is lost.

## Using Blackboard's Numerical Assessment Platform to Automate Grading Large Class Mathematics Assessments

MICHELLE ELLIS ERASMUS*

*Central University of Technology*
*mellis@cut.ac.za*

SAMS Classification: 16

In an attempt to reduce the marking load of over 700 mathematics 1 tests in the engineering department of the Central University of Technology, a portion of the test was replaced with multiple choice, but, as solution guessing amongst students was evident, an alternative avenue to multiple choice had to be found to reduce the mark load without compromising test integrity. This was accomplished by instead presenting solutions without their coefficients, for example, if a student must determine an expression for the derivative $\frac{dy}{dx}$ of the equation $y = 4 + (3x + 1)^{e^{-2x}}$, the solution is shown as $(3x+1)^{e^{-2x}} e^{q_1 x} \left( q_2 \ln(*) + \frac{q_3}{3x+1} \right) + **$. Students compare their solutions to the given format and upload the responses $q_1, q_2$ and $q_3$ onto the numeric assessment platform on Blackboard. The symbol $*$ indicates hidden terms and $**$ terms different to those already shown. If a student does not have the term $q_2 \ln(*)$ in his solution, he will choose $q_2 = 0$ otherwise he will upload the numeric coefficient value he determined in his own solution. As coefficients of terms can be zero, students are not guided as to the correctness of their own expressions which to a large extent reduces an attempt to reverse engineer solutions. Response entry together with a structured approach where incorrect responses do not affect others, was fine tuned over the past two years resulting in mark load reduction and students obtaining grades in line with assessments not containing such interventions.

## Exploring First-Year Engineering Students' Prior Knowledge in Mathematics and the Connection between Early Assessment Results

RINA DURANDT*

*University of the Witwatersrand*
*rina.durandt@wits.ac.za*

SAMS Classification: 16

The transition between education levels is a main theme in mathematics education, particularly the secondary-tertiary transition (STT), and is often associated with difficulties in students' learning of mathematics. One reason for this situation is many students lack basic skills and abilities and have a constricted disposition towards mathematics. This paper reports on empirical results from a study conducted early in 2022 at a large public university in Johannesburg concerning the prior mathematical knowledge of first-year engineering students. 276 students in a mainstream programme were exposed to a carefully designed entrance test that consists of different content and knowledge components mainly from school mathematics. The investigation is meant to yield interesting results with the aim to determine: (i) the students' performance level in the beginning, (ii) possible knowledge gaps that might highlight the necessity for additional strategic support for their future mathematics courses, and (iii) the connection between prior knowledge and early assessment results.

## Solutions and conserved vectors of the Gilson-Pickering equation

ANILA MEHMOOD*

*North-West University*
*anilamehmood1985@gmail.com*

CHAUDRY MASOOD KHALIQUE

*North-West University*
*Masood.Khalique@nwu.ac.za*

SAMS Classification: 03

In this talk, we invoke Lie group analysis [1–4] on a third-order nonlinear partial differential equation, namely the Gilson-Pickering equation [5]. We first compute Lie symmetries and then perform symmetry reductions on it. Thereafter, we use the Kudryashov's method and $(G'/G)$–expansion method to obtain its explicit exact solutions. Furthermore, conserved vectors for this equation are derived using the multiplier method.

References:

[1] G.W. Bluman, A.F. Cheviakov, S.C. Anco, Applications of Symmetry Methods to Partial Differential Equations, Springer, New York, 2010.

[2] N.H. Ibragimov, Elementary Lie Group Analysis and Ordinary Differential Equations, John Wiley & Sons, Chichester, NY, 1999.

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## Travelling wave solutions and conservation laws of the new generalized 2D Korteweg-de Vries equation

BOIKANYO PRETTY SEBOGODI*

*North-West University*
*boikanyosebogodi@gmail.com*

CHAUDRY MASOOD KHALIQUE

*North-West University*
*Masood.Khalique@nwu.ac.za*

SAMS Classification: 03

In this talk, we perform Lie group analysis [1–4] on a third-order nonlinear partial differential equation, namely the new generalized (2+1)-dimensional Korteweg-de Vries equation [5]. We first compute Lie symmetries and then perform symmetry reductions on it. Thereafter, we use the direct integration method, the simplest equation method and Kudryashov's method to obtain its travelling wave solutions. Furthermore, conserved vectors for this equation are derived using the multiplier method [4].

References:

[1] G.W. Bluman, A.F. Cheviakov, S.C. Anco, Applications of Symmetry Methods to Partial Differential Equations, Springer, New York, 2010.

[2] N.H. Ibragimov, Elementary Lie Group Analysis and Ordinary Differential Equations, John Wiley & Sons, Chichester, NY, 1999.

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## On the conservation laws and solutions of the new 2D Boussinesq equation

CHRISTINA MAJOLA*

*North-West University*
*Christina.Majola@nwu.ac.za*

LETLHOGONOLO DADDY MOLELEKI

*North-West University*
*Letlhogonolo.Moleleki@nwu.ac.za*

CHAUDRY MASOOD KHALIQUE

*North-West University*
*Masood.Khalique@nwu.ac.za*

SAMS Classification: 03

In this talk, a new 2D Boussinesq equation [1] is studied using Lie symmetry analysis technique [2-3]. The symmetry reductions are carried out and new exact solutions are obtained using the $(G'/G)$–expansion method and the Kudryoshov's method. Furthermore, the conservation laws are derived using the general multiplier method [3].

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[1] A.M. Wazwaz, L. Kaur, New integrable Boussinesq equations of distinct dimensions with diverse variety of soliton solutions, Nonlinear Dyn., 97 (2019) 83–94.

[2] G.W. Bluman and S. Kumei, Symmetries and Differential Equations, Applied Mathematical Sciences, 81, Springer-Verlag, New York, (1989).

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**A study of a generalized Zakharov-Kuznetsov equation with dual power-law nonlinearity**

CHAUDRY MASOOD KHALIQUE*

*North-West University*
*Masood.Khalique@nwu.ac.za*

OKE DAVIES ADEYEMO

*North-West Unviersity*
*adeyemodaviz@gmail.com*

In this talk we present a study on a generalized Zakharov-Kuznetsov equation with dual power-law nonlinearity arising in engineering and nonlinear science. We obtain exact solutions for the underlying equation via Lie group approach as well as direct integration method. Moreover, we construct conserved currents of the aforementioned equation via Noether's theorem with Helmholtz criteria and multiplier technique through the homotopy formula.

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[1] N.H. Ibragimov, Elementary Lie Group Analysis and Ordinary Differential Equations, John Wiley & Sons, Chichester, NY, 1999

[2] G.W. Bluman, S. Kumei, Symmetries and Differential Equations, Springer- Verlag, New York, 1989

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[4] N.H. Ibragimov, CRC Handbook of Lie Group Analysis of Differential Equations, Vols 1-3, CRC Press, Boca Raton, Florida, 1994-1996

## Boundary layer flow in convergent and divergent channels with suction or blowing at the boundary

D P MASON*

*University of the Witwatersrand*
*David.Mason@wits.ac.za*

M BOLOKA

*University of the Witwatersrand*

SAMS Classification: 03

The Lie point symmetries of Prandtl's two-dimensional boundary layer equations expressed in terms of the stream function are derived. The general form of the invariant solutions and boundary conditions, which include slip, suction and blowing at the boundary are obtained. The analytical solutions for boundary layer flow in convergent and divergent channels generated by Lie point symmetries which are not scaling symmetries are investigated. The effect of suction and blowing with no slip at the boundaries is investigated.

## Lie symmetry analysis of MHD mixed convection flow in a porous filter chamber with chemical reaction and heat source/sink

MODISAWATSONA LUCAS LEKOKO

*North-West University*
*lucaslekoko@gmail.com*

GABRIEL MAGALAKWE*

*North-West University*
*Gabriel.Magalakwe@nwu.ac.za*

SAMS Classification: 03

There are two modes of operation for industrial filtration design/equipment, namely constant rate and pressure filtration. Some industrial filter designs can operate on both constant rate and pressure filtration [1.2]. Therefore, the reference point of the current filter design is to look at the filter design that operates under low differential pressure to allow the MHD fluid to flow through the filter medium embedded inside the horizontal weakly dilating channel. The study analyses the physical interpretation of the internal mass and mixed convection flows under the influence of constant magnetic field strength, radiation, chemical reaction and heat source/sink. The suggested fluid dynamics model inside the filter chamber obeys the conservation equations in the form of partial differential equations, which are transformed into ordinary differential equations by employing the Lie group analysis [3,4]. The approximate analytical solutions of the study are obtained using the perturbation method [5,6]. The excellent agreement of the internal fluid velocity, temperature and concentration profiles were verified with the numerical solutions obtained using the NDSolve Mathematica package. The influences of the parameters on the internal flow properties are illustrated through graphs and tables. Results reveal that enhancing the chemical reaction (Kr) and Schmidt number (Sc) decreases the internal fluid concentration while amplifying the skin friction coefficient and Sherwood number. A rise in local heat transfer at the bottom wall is observed when the buoyancy ratio (N) and Richardson number (Ri) increase, which consequently minimises the skin friction coefficient.

References:

[1] C. R. O'Melia, Particles, pretreatment, and performance in water filtration, J. Environ. Eng., 111 (6) (1985), 874-890.

[2] E. Iritani, M. Kostic, A review on modeling of pore-blocking behaviors of membranes during pressurized membrane filtration, Dry. Technol., 31 (2) (2013), 146-162.

[3] G. Magalakwe, M. L. Lekoko, K. Modise, and C. M. Khalique, Lie group analysis for MHD squeezing flow of viscous fluid saturated in porous media, Alex. Eng. J., 58(3) (2019), 1001-1010.

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[7] M. L. Lekoko, G. Magalakwe, Parametric analysis on MHD mixed convection flow in a porous filter chamber with chemical reaction and heat source/sink: analytical approach, Pramana, 97(4) (2023), 154.

[8] G. W. Bluman, S. Kumei, Symmetries and differential equations, Springer Science & Business Media, (Vol. 81) 2013.

## Symmetry reductions, exact solutions and conserved vectors of a modified (3+1)-dimensional Zakharov-Kuznetsov equation

GONTSE SAMSON PAI*

*North-West University*
*gontsepai@gmail.com*

CHAUDRY MASOOD KHALIQUE

*North-West University*
*Masood.Khalique@nwu.ac.za*

SAMS Classification: 03

In this presentation, we delve into the intricate realm of nonlinear propagation phenomena concerning dust-ion acoustic solitary waves and shocks. These fascinating dynamics are encapsulated by a nonlinear evolution partial differential equation, specifically the perturbed (3+1)-dimensional Zakharov-Kuznetsov (pZK) equation. Our primary focus revolves around the augmentation of this equation, leading us to explore the intricacies of a (3+1)-dimensional nonlinear modified pZK equation. Within this context, we construct an optimal system and meticulously perform symmetry reductions, ultimately unveiling a collection of exact invariant solutions. To further deepen our understanding of the modified pZK equation's underlying principles, we invoke Noether's theorem in conjunction with a predefined Lagrangian. This strategic approach empowers us to elucidate and establish a comprehensive set of conservation laws governing the dynamics of this modified equation. Our findings promise to shed valuable light on the intricate behavior of dust-ion acoustic solitary waves and shocks within the realm of nonlinear physics.

References:

[1] N.H. Ibragimov, Elementary Lie Group Analysis and Ordinary Differential Equations, John Wiley & Sons, Chichester, NY, 1999.

[2] L.V. Ovsiannikov, Group Analysis of Differential Equations Moscow Nauka 1978. In published by Academic Press. New York. (1982).

[3] J.P. Olver, Applications of Lie groups to differential equations, Springer Science & Business Media. 107 (1993).

## Conservation laws and solutions of the generalised modified Camassa-Holm equation

ISAAC KEOAGILE MOHAPI*

*North-West University*
*isaackeoagile@gmail.com*

CHAUDRY MASOOD KHALIQUE

*North-West University*
*Masood.Khalique@nwu.ac.za*

SAMS Classification: 03

In this talk, we investigate the general modified Camassa-Holm equation [1], which has many applications in fields such as physics and other related science fields. Using the technique of Lie symmetry analysis [2], we first compute its Lie point symmetries. Thereafter, group-invariant solutions are determined under each symmetry. Finally, conservation laws for this equation are derived using multiplier approach [3].

References:

[1] A. Zulfiqar, J. Ahmad, Exact solitary wave solutions of fractional modified Camassa-Holm equation using an efficient method, Alex. Eng. J., 59 (2020) 3565–3574.

[2] L.V. Ovsiannikov, Group Analysis of Differential Equations, Academic Press, New York, 1982.

[3] P.J. Olver, Applications of Lie Groups to Differential Equations, Springer-Verlag, New York, 1993.

## First integrals, solutions and conservation laws of the derivative nonlinear Schrödinger equation

KARABO PLAATJIE*

*North-West University*
*25451308@nwu.ac.za*

CHAUDRY MASOOD KHALIQUE

*North-West University*
*Masood.Khalique@nwu.ac.za*

SAMS Classification: 03

The derivative nonlinear Schrödinger equation which has many applications, such as the propagation of circular polarised nonlinear Alfvén waves in plasmas, will be studied. In this talk, using first integrals, we present general and special solutions of this equation. Classical Lie theory will be applied to compute exact analytical solutions of this equation. Furthermore, using multiplier method, conservation laws of the underlying equation will be constructed.

References:

[1] V.M. Lashkin, N-soliton solutions and perturbation theory for the derivative nonlinear Schrödinger equation with nonvanishing boundary conditions, J. Phys. A: Math. Theor., 40 (2007) 6119–6132.

[2] L.V. Ovsiannikov, Group Analysis of Differential Equations, Academic Press, New York, 1982.

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## On the solutions and conserved vectors of a 3D generalized BKP-Boussinesq equation

LETLHOGONOLO DADDY MOLELEKI*

*North-West University*
*Letlhogonolo.Moleleki@nwu.ac.za*

CHAUDRY MASOOD KHALIQUE

*North-West University*
*Masood.Khalique@nwu.ac.za*

SAMS Classification: 03

In this talk, a (3+1)-D generalized B-type KP-Boussinesq equation [1], which was recently formulated in the literature, is investigated here from Lie group standpoint [2–4]. A solution is obtained by Lie symmetry reductions and direct integration in terms of incomplete elliptic integral. Furthermore, hyperbolic and trigonometric functions solutions are derived by invoking the $(G'/G)$–expansion method. Finally, we construct conservation laws of the aforementioned equation by utilizing the conservation theorem due to Ibragimov.

References:

[1] A.M. Wazwaz, S. El-Tantawy, Solving the (3+1)-dimensional KP-Boussinesq and BKP-Boussinesq equations by the simplified Hirota's method. Nonlinear Dyn. 2017;88:3017–21.

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## Lie group analysis and conserved vectors of the generalized (3+1)-dimensional KP-BBM equation

BODIBE JONATHAN LEBOGANG*

*North-West University*
*lebo110812@gmail.com*

CHAUDRY MASOOD KHALIQUE

*North-West University*
*Masood.Khalique@nwu.ac.za*

SAMS Classification: 03

In this talk, we present Lie group analysis of the nonlinear (3+1)-dimensional Kadomtsev-Petviashvili Benjamin-Bona-Mahony equation. We find exact solutions of the equation using Lie symmetry method together with Kudryashov's and $(G'/G)$-expansion methods. Moreover, we derive the conservation laws for the equation using the Ibragimov's methods.

References:

[1] P.J. Olver, Applications of Lie Groups to Differential Equations, second ed., Springer-Verlag, Berlin, 1993.

[2] P.E. Hydon, Symmetry Methods for Differential Equations, Cambridge University Press, Cambridge, NY, 2000.

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**Lie symmetries, exact solutions and conservation laws of a 3D Gardner-type equation.**

MASEGO MAFORA*

*North-West University*
*masegomafora07@gmail.com*

LETLHOGONOLO DADDY MOLELEKI

*North-West University*
*Letlhogonolo.Moleleki@nwu.ac.za*

GABRIEL MAGALAKWE

*North-West University*
*Gabriel.Magalakwe@nwu.ac.za*

SAMS Classification: 03

In this talk we investigate the nonlinear partial differential equation (NPDE) model, namely, the (3+1)-dimensional Gardner-type equation [1], which models sound waves in nonlinear physical phenomena. We use the Lie symmetry method [2] to transform the NPDE model that represents sound waves into nonlinear ordinary differential equation (NODE) without changing the dynamics of the model. Exact solutions are obtained using different techniques to solve the NODE. Moreover, conserved vectors are derived using Ibragimov's theorem [3].

References:

[1] A.M. Wazwaz, Two (3+1)-dimensional Gardner-type equation with multiple kink solutions, Rom. Rep. Phys., 69 (2017) 108.

[2] L.V. Ovsiannikov, Group Analysis of Differential Equations, Academic Press, New York, (1982).

[3] N.H. Ibragimov, A new conservation theorem, Journal of Mathematical Analysis and Applications, 333(1) (2007) 311–328.

## Applications of Lie symmetries to heat conduction in a longitudinal fin of concave parabolic profile

MFANAFIKILE DON MHLONGO*

*University of KwaZulu-Natal*
*mhlongomd@ukzn.ac.za*

BENJAMIN FRANKLIN SHOZI

*Mangosuthu University of Technology*
*franklin@mut.ac.za*

VUSI ANDILE BULUNGA

*University of KwaZulu-Natal*
*vusibulunga@gmail.com*

SAMS Classification: 03

In this paper we study heat conduction in a longitudinal fin of concave parabolic profile. The thermal conductivity and heat transfer coefficient vary nonlinearly with temperature. The step change in base temperature and step change in base heat flow were used as boundary conditions. We employed Lie symmetry techniques in an attempt to solve initial and boundary value problem. The governing equation admitted two local symmetries which reduced the original non-linear partial differential equation (PDE) into two non-linear ordinary differential equations (ODEs). One of these did not satisfy the step change in temperature conditions. By using method of differential invariants the non-linear ODEs were further reduced into two first-order ODEs of Abel's second kind. At this stage we considered a special case when m=0. This yielded an exact analytical solution which enabled us to carry out further analysis. On the other hand the second equation was transformed into canonical form; the results are given in special function. Further application of the method of differential invariant to the Abel's equation yielded exact analytical solutions in the form of implicit functions. These were analyzed for the behavior of temperature and heat flux in term of increasing space, time, thermo-geometric fin parameter.

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## Analytic solutions and conservation laws of the classical Lonngren wave equation for communication signals

MDUDUZI YOLANE THABO LEPHOKO*

*North-West University*
*dasmice@gmail.com*

CHAUDRY MASOOD KHALIQUE

*North-West University*
*Masood.Khalique@nwu.ac.za*

SAMS Classification: 03

This talk undertakes a comprehensive examination of the classical Lonngren wave equation [1], a fundamental computational model used for simulating electrical signals in semiconductor materials, with specific emphasis on the tunnel diode. It marks the first instance in which the classical Lonngren wave equation is subjected to analysis via Lie symmetries [2], along with other specialized techniques including the power series method and Jacobi elliptic expansion technique. Furthermore, the multiplier method [3] is applied to derive conserved vectors.

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## Invariants and closed-form solutions with conservation laws of a nonlinear multi-dimensional partial differential equation

OKE DAVIES ADEYEMO*

*North-West University*
*adeyemodaviz@gmail.com*

CHAUDRY MASOOD KHALIQUE

*North-West University*
*Masood.Khalique@nwu.ac.za*

SAMS Classification: 03

In this talk, we present the analytical examination of a nonlinear multi-dimensional equation [1]. We first apply Lie group analysis [2,3] to obtain various infinitesimal generators admitted by the equation. This is followed by invoking the generators to reduce the understudy equation to achieve copious group-invariant solutions. Thus, various closed-form solutions of interest were obtained for the underlying equation. Besides, we construct the conservation laws [3,4] of the equation.

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## Unsteady slender rivulet flow down an inclined porous plane

M R R KGATLE-MASEKO*

*University of the Witwatersrand*
*Rahab.Kgatle@wits.ac.za*

D P MASON

*University of the Witwatersrand*
*David.Mason@wits.ac.za*

A E R LOWRY-CORRY

*University of the Witwatersrand*

SAMS Classification: 03

The unsteady three-dimensional flow of a thin slender rivulet of incompressible Newtonian fluid down an inclined porous plane is investigated. A second order nonlinear partial differential equation in two spatial variables and time and containing the leak-off velocity is derived for the height of the thin slender rivulet. Lie point symmetries of the model rivulet equation are calculated and used to determine the group invariant solution for the height of the rivulet. The leak-off velocity is not specified a priori but is determined in the process of the derivation of the group invariant solution. Using Lie group analysis it is found that the partial differential equation can be reduced in two steps to an ordinary differential equation provided the leak-off velocity satisfies a first order linear partial differential equation in three variables. An exact analytical solution with a dry patch in the central region is derived for a special leak-off velocity. Numerical solutions for the height of the rivulet are obtained using a shooting method. Three models of the leak-off velocity are analysed. In the first model, the leak-off velocity is proportional to the height of the rivulet; in the second, it is proportional to the square of the height and in the third, it is proportional to the cube of the height.

## Noether Symmetries of a Generalized Coupled Lane-Emden-Klein-Gordon-Fock System with Central Symmetry

SIVENATHI OSCAR MBUSI*

*North-West University*
*Sivenathi.Mbusi@nwu.ac.za*

BEN MUATJETJEJA

*University of Botswana*
*ben.muatjetjeja@mopipi.ub.bw*

ABDULLAHI RASHID ADEM

*University of South Africa*
*ademar@unisa.ac.za*

SAMS Classification: 03

In this talk we carry out a complete Noether symmetry analysis of a generalized coupled Lane-Emden-Klein-Gordon-Fock system with central symmetry [1-2]. It is shown that several cases transpire for which the Noether symmetries exist. Moreover, we derive conservation laws connected with the admitted Noether symmetries. Furthermore, we fleetingly discuss the physical interpretation of the these conserved vectors.

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## Solutions of injection driven flow and heat transfer inside an inclined chamber: Symmetry analysis approach

MODISAWATSONA LUCAS LEKOKO

*North-West University*
*lucaslekoko@gmail.com*

GABRIEL MAGALAKWE

*North-West University*
*Gabriel.Magalakwe@nwu.ac.za*

TANKI MOTSEPA*

*University of Mpumalanga*
*Tanki.Motsepa@ump.ac.za*

SAMS Classification: 03

In this study, we find closed-form solutions of momentum and temperature variation during the unsteady state filtration process to advance fluid purification [1]. Lie group analysis is used to transform a system of equations representing the flow and heat transfer [2] into a solvable system without changing the dynamics of the case study. The transformed solvable system is then solved to find closed-form solutions of momentum and temperature variation [3]. The obtained closed-form solutions [4] are then used to analyse the effects of physical parameters arising from the process dynamics to find combinations of parameters that yield maximum permeates outflow [5]. The results indicate that as time evolves, permeates production increases because internal momentum, work done, and energy intensifies with time. In addition, to increase permeates production, high injection rate, permeates wave speed parameter, permeation, angle of inclination and Grashof number enhance permeates outflow.

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## Collision properties of nonlinear waves in the modified Gardner equation

WENDY MANGANYE*

*North-West University*
*wmanganye1@gmail.com*

SAMS Classification: 23

Small-amplitude ion-acoustic waves near a supercritical plasma compositions are described by modified Gardner equation [1-3]. Previous studies revealed parametric existence regions for supersoliton in the modifed Gardner equation and showed that collisions between solitons and supersolitons [3]. This study aims to extend existing results. We analyze solutions of the modified Gardner equation and simulate collisions for soliton-type solutions in this work. This is accomplished through the application of a moving frame analysis and that reduces the modified Gardner equation to a first order ordinary differential equation. This is evaluated using roots of related cubic polynomial analysis that helps in identifying coefficients where interesting nonlinear solutions occur such as double layers and coexistence of positive and negative solitons and supersolitons occur . This problem is then solved numerically by means of fourth-order Runge-Kutta method using Matlab code. We performed numerical simulations for solitons overtaking nonlinear waves. The simulations show that double layers maintains their shape and speed after collision, where as solitons that existed before collision is replaced by the one that has a opposite polarity. These results may be useful in deepening the understanding of nonlinear waves in space plasmas.

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## Group-invariant and explicit analytic solutions, and conservation laws of the generalized three-dimensional KP-BBM equation of mathematical physics

YANGA GAXELA*

*North-West University*
*gaxelayanga@gmail.com*

CHAUDRY MASOOD KHALIQUE

*North-West University*
*Masood.Khalique@nwu.ac.za*

SAMS Classification: 03

This paper examines a high-dimensional nonlinear partial differential equation called the generalized Kadomtsev-Petviashvili-Benjamin-Bona-Mahony (KP-BBM) equation existing in three dimensions [1]. The Lie symmetry analysis of the equation is carried out step-by-step [2]. In consequence, we found symmetries from which various groupinvariant solutions results from which numerous solutions of interest that satisfy the KP-BBM equation are obtained. Solutions of interest secured include hyperbolic functions as well as elliptic functions with the latter being the more general of the two solutions. Besides, a good number of algebraic solutions with arbitrary functions are also achieved [3]. Moreover, the dynamics of the solutions are further explored diagrammatically using computer software. In the concluding part, various conservation laws of the underlying model are constructed via the multiplier method as well as the Noether theorem [4].

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## A group theoretic proof of the existence of the Plancherel weight

L E LABUSCHAGNE*

*North-West University*
*louis.labuschagne@nwu.ac.za*

SAMS Classification: 11, 24, 33

Given a locally compact group $G$ we write $VN(G)$ for the group von Neumann algebra generated by the left-shift operators $\lambda_s$ ($s \in G$) on $L^2(G)$. An element $f \in L^2(G)$ is said to be *left-bounded* if the convolution operation $g \to f * g$ (valid on functions $g$ of compact support) extends to a bounded operator $\lambda_f$ on $L^2(G)$. Such operators still belong to $VN(G)$. It is a well known fact that the prescription of setting $\psi_G(x^*x)$ equal to $\int_G |f|^2\,dt$ if $x = \lambda_f$ for some left-bounded $f \in L^2(G)$ and equal to $\infty$ otherwise, defines a faithful normal semifinite weight on $VN(G)$ - the so-called *Plancherel weight*. All known proofs of this fact valid for general LCG's, pass through the theory of left Hilbert algebras. We present a direct proof of the existence of the Plancherel weight valid for the most general LCG's using only group theoretic arguments. The formulae developed in the proof then further enable us to give a simple proof of the fact that the Plancherel weight is finite if and only if the group is discrete.

## Examples of type $\mathrm{III}_\lambda$ group von Neumann algebras

CHRISTIAAN H. PRETORIUS*

[1] *North-West University*
[2] *DSI-NRF Centre of Excellence in Mathematical and Statistical Sciences (CoE-MaSS)*
*christiaanhendrikpj@gmail.com*

SAMS Classification: 24

In 1933, Alfred Haar discovered the existence of a left (resp. right) translation invariant measure on any locally compact group $\mathrm{G}$ [1]. In his honour, such measures are called left (resp. right) Haar measures. These measures give access to a sensible theory of integration, as well as the Hilbert space $\mathrm{L}^2(\mathrm{G})$. The unitary operators defined by $(\lambda(\mathrm{g})\xi)(\mathrm{k}) = \xi(\mathrm{g}^{-1}\mathrm{k})$ for all $\mathrm{g} \in \mathrm{G}$ and $\xi \in \mathrm{L}^2(\mathrm{G})$, generate a von Neumann algebra, called a group von Neumann algebra, denoted $\mathrm{VN}_\mathrm{l}(\mathrm{G})$. We will assume that $\mathrm{G}$ is also separable. In this case, we are able to make use of some structure theorems (derived by Colin E. Sutherland [2]) that relates $\mathrm{G}$ to $\mathrm{VN}_\mathrm{l}(\mathrm{G})$. Using said structure theorems, examples of type $\mathrm{III}_\lambda$, $\lambda \in (0, 1]$ group von Neumann algebras are constructed [2].

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## Commutativity and orthogonality of similarity orbits in Banach algebras

R. BRITS

*University of Johannesburg*
*rbrits@uj.ac.za*

M. HASSEN

*University of Johannesburg*
*mhassen@uj.ac.za*

F. SCHULZ*

*University of Johannesburg*
*francoiss@uj.ac.za*

SAMS Classification: 30

Let $\mathrm{A}$ be a unital semi-simple Banach algebra over $\mathbb{C}$, and denote by $\mathrm{G}_1(\mathrm{A})$ the principal component of the invertible group of $\mathrm{A}$. The similarity orbit of any $\mathrm{a} \in \mathrm{A}$ is then defined to be $\mathrm{orb}(\mathrm{a}) := \{\mathrm{waw}^{-1} : \mathrm{w} \in \mathrm{G}_1(\mathrm{A})\}$. In this talk we shall prove that the similarity orbits $\mathrm{orb}(\mathrm{a})$ and $\mathrm{orb}(\mathrm{b})$ for $\mathrm{a}, \mathrm{b} \in \mathrm{A}$ commute precisely when there is at least one complex number which does not belong to the spectrum of any of the products $\mathrm{a}'\mathrm{b}'$, where $\mathrm{a}' \in \mathrm{orb}(\mathrm{a})$ and $\mathrm{b}' \in \mathrm{orb}(\mathrm{b})$. A similar statement to the above holds if $\mathrm{a}'\mathrm{b}'$ is replaced by $\mathrm{a}'+\mathrm{b}'$, $\mathrm{a}'-\mathrm{b}'$, or $\mathrm{a}'+\mathrm{b}'-\mathrm{a}'\mathrm{b}'$. Moreover, if in addition one assumes that $\mathrm{orb}(\mathrm{a}) = \mathrm{orb}(\mathrm{b})$, then the conclusion is that $\mathrm{a}$ commutes with every element in $\mathrm{A}$. This generalizes part of an old theorem by J. Zemanék (see [3, Theorem 4.3]), and yields improvements on two classical characterizations of commutative Banach algebras. The techniques employed to establish our results make extensive use of representation theory, more specifically, of Sinclair's Density Theorem. Using the theory surrounding analytic multifunctions, it is also possible to localize our results in some sense (although this will not be the focus of the talk). To conclude, we restrict our attention to algebraic elements and idempotents in particular, and see that the orthogonality of two similarity orbits of idempotents is equivalent to a pair of spectral radius properties. This research is largely motivated by the work in [1], [2], and [3].

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## Spectral Characterizations of Idempotents in Banach Algebras

MUHAMMAD HASSEN*

*University of Johannesburg*
*mhassen@uj.ac.za*

RUDI BRITS

*University of Johannesburg*
*rbrits@uj.ac.za*

FRANCOIS SCHULZ

*University of Johannesburg*
*francoiss@uj.ac.za*

SAMS Classification: 30

Let $\mathrm{A}$ be a complex and unital Banach algebra, and let $\mathrm{E}$ denote the collection of idempotents of $\mathrm{A}$. An old paper of J. Zemánek's ([3]), which was really the starting point of studies of idempotents in general Banach algebras, exhibits a multitude of results concerning the set $\mathrm{E}$. More specifically, it was shown that the connected component of $\mathrm{E}$ which contains $\mathrm{p} \in \mathrm{E}$, say $\mathrm{E_p}$, is precisely the set of elements of the form $\mathrm{wpw^{-1}}$—where $\mathrm{w}$ runs through the principal component of the invertible group of $\mathrm{A}$. Following Zemánek's work, a considerable number of papers concerning the form of paths connecting the members of $\mathrm{E_p}$ have seen the light of day. In this talk, we elaborate on the second part of Zemánek's article which characterizes central idempotents in terms of spectral conditions. We use modern techniques and results, together with the well-known Scarcity Theorem ([1, Theorem 3.4.25]) due to B. Aupetit and our new extended version thereof which appeared in [2] to show that the global algebraic conditions which characterize central idempotents of $\mathrm{A}$ given in [3] can be replaced by significantly weaker local spectral conditions.
Specifically, this talk concerns the characterization of those $\mathrm{p} \in \mathrm{E}$ for which $\mathrm{E_p} = \{\mathrm{p}\}$. Notably, amongst our results, we show that if the spectra of products $\mathrm{pq}$, or differences $\mathrm{p-q}$, merely resembles the spectrum of idempotents for $\mathrm{q}$'s in a relatively small subset of $\mathrm{E_p}$, then in fact $\mathrm{E_p} = \{\mathrm{p}\}$.

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## Perturbation ideals and Fredholm theory in Banach algebras

TSHIKHUDO LUKOTO *

*University of Limpopo*
*tshikhudo.lukoto@ul.ac.za*

HEINRICH RAUBENHEIMER

*University of Johannesburg*
*heinrichr@uj.ac.za*

SAMS Classification: 11

Following the exploration of perturbation classes which was introduced in [2], in this talk we explore more perturbation ideals of sets that generate the familiar spectra in Fredholm theory. If $\Phi(\mathrm{I})$ is the set of Fredholm elements relative to a closed ideal $\mathrm{I}$ in a Banach algebra $\mathrm{A}$, the perturbation of this set is equal to $\pi^{-1}(\mathrm{Rad}(\mathrm{A/I}))$ where $\pi : \mathrm{A} \to \mathrm{A/I}$ is the canonical homomorphism defined by $\pi(\mathrm{x}) = \mathrm{x} + \mathrm{I}$ ($\mathrm{x} \in \mathrm{A}$) and $\mathrm{Rad}(\mathrm{A})$ is the Jacobson radical in $\mathrm{A}$. This perturbation result proved very useful in generating more perturbation results and can be found in [3]. Also of particular interest, these sets were also classified as either regularity or a semiregularity, an axiomatic framework to spectral theory in Banach algebras which was first introduced in [1].

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# Invertibility of unbounded Toeplitz operators with rational matrix symbols: Riccati equations approach

SANNE TER HORST*

*North-West University*
*sanne.terhorst@nwu.ac.za*

GILBERT J. GROENEWALD

*North-West University*
*gilbert.groenewald@nwu.ac.za*

JACOB J. JAFTHA

*University of Cape Town*
*jacob.jaftha@uct.ac.za*

ANDRÉ C.M. RAN

*VU Amsterdam & North-West University*
*a.c.m.ran@vu.nl*

SAMS Classification: 11

This paper is a continuation of the work on unbounded Toeplitz-like operators $\mathrm{T}_\Omega$ with rational matrix symbol $\Omega$ initiated in [1], where a Wiener-Hopf type factorization of $\Omega$ is obtained and used to determine when $\mathrm{T}_\Omega$ is Fredholm and compute the Fredholm index in case $\mathrm{T}_\Omega$ is Fredholm. Due to the high level of non-uniqueness and complicated form of the Wiener-Hopf type factorization, it does not appear useful in determining when $\mathrm{T}_\Omega$ is invertible [2]. In this talk we consider state space methods to characterize invertibility of $\mathrm{T}_\Omega$ in terms of the existence of a stabilizing solution of an associated non-symmetric discrete algebraic Riccati equation, which in turn leads to a pseudo-canonical factorization of $\Omega$ and concrete formulas of $\mathrm{T}_\Omega^{-1}$. The talk is based on [3].

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## Hyperbolicity of Linear Partial Differential Equations of Neutral Type

MARIEME LASRI *

*University of Free State*
*Lasri.M@ufs.ac.za*

HAMID BOUNIT

*Ibn Zohr University Morocco*
*h.bounit@uiz.ac.ma*

SAID HADD

*Ibn Zohr University Morocco*
*s.hadd@uiz.ac.ma*

SAMS Classification: 10

This present work is mainly devoted to investigate the property of spectral decomposition of neutral differential equations in infinite dimensional setting, that is the exponential dichotomy, of the following form

$$\begin{cases} \dfrac{d}{dt}(z(t) - Dz_t) = A(z(t) - Dz_t) + Lz_t, \quad t \geq 0, \\ \lim\limits_{t \mapsto 0}(z(t) - Dz_t) = x, \quad z_0 = \varphi, \end{cases} \tag{1}$$

where the operator $A : D(A) \subset X \to X$ generates a $C_0$–semigroup $(T(t))_{t \geq 0}$ and $L, D : W^{1,p}([-r, 0], X) \to X$ are given by Riemann-Stieltjes integrals.

In fact, we prove that the exponential dichotomy of the associated semigroup to such equations does not depend on that of their associated difference equations ( see, e.g., [1,3,4]). Based on the regular linear systems and feedback theory, we introduce a new transformation of neutral-type equations which plays a key role in our investigation (see for instance [2,5,6,7,8,9]).

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# $\mathcal{H}_2$-Optimal control of poset-causal systems

JACOBUS ZEELIE*

*North West University*
*jaccie.zeelie@nwu.ac.za*

SANNE TER HORST

*North West University*
*Sanne.TerHorst@nwu.ac.za*

SAMS Classification: 25

We consider decentralized linear time-invariant (LTI) systems which consist of several interconnected subsystems. The communication structure between the subsystems is reflected by a block-zero pattern in the system matrices. Such block zero patterns are determined by binary relations on the set $\mathrm{P} = \{1, \ldots, \mathrm{p}\}$ where $\mathrm{p}$ is the number of subsystems. Structured linear systems are linear systems that have an underlying binary relation that determines the block zero pattern of the system matrices. Specifically, poset causal systems have an underlying partial order that determines their structure.

The optimal control of structured linear systems proves to be challenging due to the additional structural constraint. In the classical centralized setting for the $\mathcal{H}_2$-control problem, all stabilizing controllers of a plant are given by the Youla parametrization. A reparameterized problem may then be solved in the Youla parameter from which an optimal controller can be recovered. We consider the $\mathcal{H}_2$-control problem for poset-causal systems. The state feedback case has been solved by Shah. A related reparameterized problem reduces to several local classical problems which can readily be solved via a spectral factorization approach. The output feedback case is considerably more difficult. We consider various solution strategies based on a detailed analysis of various approaches to the unstructured output feedback control problem. These approaches aim to reduce the structured control problem to multiple unstructured local problems. Although an optimal structured solution is difficult to obtain, various optimality considerations are taken into account in the construction of feasible controllers for the output feedback case.

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## Quantum Wasserstein distance of order 1 between channels

ROCCO DUVENHAGE

*University of Pretoria*
*rocco.duvenhage@up.ac.za*

MATHUMO MAPAYA*

*University of Pretoria*
*u18026312@tuks.co.za*

SAMS Classification: 11, 20, 24.

In [2] we adapt and generalise a formulation of the quantum Wasserstein distance of order 1 between states in finite dimensions developed in [1] to set up a general theory for Wasserstein distance of order 1 between the unital maps from one algebra to another. This gives us a metric on the set of unital maps from one composite system to another, which is deeply connected to the reductions of the unital maps. We use the fact that channels are unital maps with extra structure, to systematically define a quantum Wasserstein distance of order 1 between channels, i.e., a metric on the set of channels. Lastly, the additivity and stability properties of this metric are studied.

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## Towards a Banach-Stone theorem for Operator Systems

LOUIS LABUSCHAGNE

*North West University*
*louis.labuschagne@nwu.ac.za*

DYLAN PHELPS *

*North West University*
*49896113@mynwu.ac.za*

SAMS Classification: 24

The classical Banach-Stone theorem establishes a link between *-homomorphims between $\mathcal{C}(\mathrm{K}_0)$ and $\mathcal{C}(\mathrm{K}_1)$ and continuous maps between $\mathrm{K}_0$ to $\mathrm{K}_1$. In the non-commutative C*-algebra context $\mathcal{C}(\mathrm{K})$ spaces are replaced by C*-algebras, and compact Hausdorff spaces are replaced by pure state spaces. In [1] and [2] a classification of maps which preserve pure states is obtained as a non-commutative version of the Banach-Stone theorem. We aim to obtain a similar result for operator systems by considering boundary representations in place of pure states.

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## Category measures, the dual of $\mathrm{C(K)}^{\delta}$ and hyper-Stonean spaces

JAN HARM VAN DER WALT

*University of Pretoria*
*janharm.vanderwalt@up.ac.za*

SAMS Classification: 11

For a compact Hausdorff space $\mathrm{K}$, we give descriptions of the dual of $\mathrm{C(K)}^{\delta}$, the Dedekind completion of the Banach lattice $\mathrm{C(K)}$ of continuous, real-valued functions on $\mathrm{K}$. We characterize those functionals which are $\sigma$-order continuous and order continuous, respectively, in terms of Oxtoby's category measures. As applications, we give a purely topological characterization of hyper-Stonean spaces, and characterize those spaces $\mathrm{K}$ for which $\mathrm{C(K)}$ admits a strictly positive order continuous functional.

## An order-theoretic abstraction of the complex modulus

LUAN NAUDE*

*University of Pretoria*
*luan.naude98@gmail.com*

SAMS Classification: 11

Traditionally, the complex modulus and other related concepts in the real and complex numbers are generalized to vector spaces using norms. But there are many vector spaces that have a natural order, yet no natural norm. In this talk, I will explain how we can reconsider these concepts in an order-theoretic way that allows them to be generalized to vector lattices. This talk will be accessible to a general audience.

## On the weak $\mathrm{p}$-consistent property in Banach lattices

ELROY ZEEKOEI*

*North-West University*
*elroy.zeekoei@nwu.ac.za*

SAMS Classification: 11

In this talk we are going to discuss weak $\mathrm{p}$-summable sequences in Banach lattices. Since it is not true in general that if $\mathrm{x_n} \xrightarrow[\infty]{\mathrm{n}} 0$ weakly, then $|\mathrm{x_n}| \xrightarrow[\infty]{\mathrm{n}} 0$ weakly, that is, in general, the lattice operations are not necessarily weakly sequentially continuous, we introduce the weak $\mathrm{p}$-consistent property to show when the lattice operations are indeed weakly sequentially continuous. We then show that an AM-space is weak $\mathrm{p}$-consistent.

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## Orthogonal polynomials on the unit circle defined by the fractional Gaussian noise

SAFARI MUKERU*

*University of South Africa*
*mukers@unisa.ac.za*

SAMS Classification: 11, 26

This talk will discuss the class of orthogonal polynomials on the unit circle (in the complex plane) with respect to the spectral measure of the discrete fractional Gaussian noise. The fractional Gaussian noise generalises the classical Gaussian white noise in the sense that it has an interesting dependence structure that vanishes asymptotically at infinity. The corresponding spectral measure is absolutely continuous. The discussion will focus on some interplays between the Verblunski coefficients, the inverse Szegö function and the inverse of the covariance matrix. More background is given in [1], [2], [3].

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## An interlacing result for Hermitian matrices in Minkowski space

M. VAN STRAATEN*

*North-West University*
*madelein.vanstraaten@nwu.ac.za*

D.B. JANSE VAN RENSBURG

*North-West University*
*dawie.jansevanrensburg@nwu.ac.za*

A.C.M. RAN

*VU Amsterdam*
*a.c.m.ran@vu.nl*

SAMS Classification: 11

The well known interlacing problem is studied, but here we consider the result for Hermitian matrices in the Minkowski space, an indefinite inner product space with one negative square. More specific, we consider the $n \times n$ matrix $A = \begin{bmatrix} J & u \\ -u^* & a \end{bmatrix}$ with $a \in \mathbb{R}$, $J = J^*$ and $u \in \mathbb{C}^{n-1}$. Then $A$ is $H$-selfadjoint with respect to the matrix $H = I_{n-1} \oplus (-1)$. The canonical form for the pair $(A, H)$ plays an important role and the sign characteristic coupled to the pair is also discussed. Inspired by some of the results in the paper [1].

References:

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# Stationary and nonstationary iterative methods for nonexpansive typemappings in Banach spaces

RAHUL SHUKLA

*Walter Sisulu University*
*rshukla.vnit@gmail.com*

RAJENDRA PANT*

*University of Johannesburg*
*rpant@gmail.com*

ADRIAN PETRUSEL

*Babes-Bolyai University, Cluj-Napoca, Romania*
*petrusel@math.ubbcluj.ro*

SAMS Classification: 11

We present stationary and nonstationary methods to approximate fixed points of a general class of nonexpansive type mappings in Banach spaces. We also present some illustrative examples and demonstrate the convergence behaviour of the nonstationary methods. Finally, we pose an open problem.

## Common Attractors of Generalized Iterated Function Systems in G-Metric Spaces

VULEDZANI MAKHOSHI*

*University of South Africa*
*vuledzani.makhoshi@univen.ac.za*

TALAT NAZIR

*University of South Africa*
*talatn@unisa.ac.za*

MELUSI KHUMALO

*University of South Africa*
*khumam@unisa.ac.za*

SAMS Classification: 11

In this presentation, we generate a novel common attractor [1] using a finite set of generalized contractive mappings that belong to a certain class of mappings defined on a G-metric space [2]. Moreover, different results for G-iterated function systems which satisfy a variety of generalized contractive conditions are acquired and validated through an illustrative example of G-iterated function system that satisfy a distinct set of generalized contractive conditions [3]. We conclude with the study of well-posedness of attractor base problems of generalized Hutchinson contractive operators [4] in G-metric spaces. The present findings demonstrate a capacity to generalize, integrate, and extend a range of conclusions observed in contemporary scholarly works.

References:

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# Strong convergence analysis for solving quasi-monotone variational inequalities and fixed point problems in reflexive Banach spaces

A.O.E. OWOLABI *

*University of Kwazulu-Natal*
*218086824@stu.ukzn.ac.za*

O.T. MEWOMO

*University of Kwazulu-Natal*
*mewomoo@ukzn.ac.za*

J.C. YAO

*China Medical University, Taichung, Taiwan*
*yaojc@mail.cmu.edu.tw*

X. QIN

*Kyung Hee University, Seoul, Korea*
*qxlxajh@163.com*

SAMS Classification: 11

In this paper, we propose a modified inertial hybrid Tseng's extragradient algorithm with self-adaptive step size for approximating a common solution of variational inequalities with quasimonotone operator and fixed point problems of a finite family of Bregman quasi-nonexpansive mappings in real reflexive Banach spaces. By using Bregman distance approach, we prove a strong convergence result for finding common solutions of these problems under some appropriate conditions on the control parameters. Our algorithm is of self-adaptive step size which generates a non-monotonic sequence of step sizes. Unlike the existing results in the literature, our algorithm does not require any linesearch technique which uses inner loops and might consume additional computational time for determining the step size. Finally, we present some numerical examples to illustrate the efficiency of our algorithm in comparison with related methods in the literature.

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## Solutions of Riesz-Caputo fractional differential equations involving Langevin equation

HEMANT KUMAR NASHINE*

*Bhopal University, India*
*hemantkumar.nashine@vitbhopal.ac.in*

In this talk, we look into the existence and uniqueness of fractional differential equations of the Langevin type. The principle of Banach's contraction is used to prove uniqueness, and Schaefer and Krasnoselskii's fixed point theorems are used to prove existence. We illustrate our methods with a few examples.

## Generalised Lambert Transforms: Mathematical Contributions of Joe Mokoena

EDER KIKIANTY*

*University of Pretoria*
*eder.kikianty@up.ac.za*

LOYISO NONGXA

*University of the Witwatersrand*
*loyiso.nongxa@wits.ac.za*

SAMS Classification: 11

Joseph Albert Mashite (Joe) Mokoena was the first black South African to be awarded a doctoral in mathematics. He received his PhD from the University of the Witwatersrand in 1958. He was awarded the Order of Ikhamanga in Gold, for his exceptional contribution to the field of mathematics, and dedication to the development of South Africa and the African continent. However, not much is known about his contribution to mathematics. In this talk, I will discuss his work on generalised Lambert transforms which was the subject of his doctoral dissertation. His work was largely based on the Lambert transform as introduced by D.V. Widder in his 1947 paper entitled “The inversion of a generalized Laplace transform”. I will also compare Mokoena’s work with known generalisations of Lambert transforms in the literature.

## Competitive stock exchanges in the financial market: Simulation of profits achieved by investors

MASIALA MAVUNGU*

*University of Johannesburg*
*msmvp7219@gmail.com*

SAMS Classification: 10

The main concern of this paper is to develop mathematical models and computational simulations governing the dynamics of five stock exchanges by some investors operating in the same financial stock market. The investors belong to the same network financial system and are competitively exchanging stocks in the market where all of them have the same licence type to trade the same type of stock in the financial market and each of them is subject to trade convention laws and regulations, competition and uncertainties. For every investor operating in the market and involved in the competition, the rate of profit is modelled as a stochastic differential equation. The set combining all the equations gives a system of stochastic differential equations'which is solved to obtain the investors profit functions.

To solve the system the following approach is used: Firstly, a Matlab function is developed to evaluate and code the system of stochastic differential equations. Secondly, a Matlab function is developed to code a Stochastic Euler-Maruyama numerical method which is used to solve the system. A third Matlab function is developed as a script function to regulate the interactions between the first two Matlab functions. The solution of the system gives the profit functions of the investors.

## Optimal Impulse Control for Bank Capital Buffer

MMBONISENI MULAUDZI*

*University of South Africa*
*mulaump@unisa.ac.za*

SAFARI MUKERU

*University of South Africa*
*mukersjm@unisa.ac.za*

SAMS Classification: 25

This study examines a bank that has the ability to manage its capital buffer by investing in both illiquid assets (loans) and liquid assets (stocks and treasuries), as well as by paying dividends and raising new capital (recapitalisation). In the sequel, the bank is required to pay both fixed and proportional costs related to dividends payment and recapitalisation. To our knowledge, there is no work on optimisation of bank capital buffer that incorporates costs associated with dividends payment. Therefore, this work extends and complements previous literature related to the topic (see [1], [2], [3], [4], [5] and [6]). Until liquidation time, the bank's goal is to maximize the expected discounted value of dividends payment, less cost of recapitalisation, by determining the optimal policy for recapitalisation, dividends and liquid assets investments in a single optimisation model.

References:

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## Cross-diffusion effects on the double-diffusive convection in a rotating vertical porous cylinder with vertical throughflow

SAMAH A. ALI*

*University of KwaZulu-Natal*
*samahanwarali@gmail.com*

PRECIOUS SIBANDA

*University of KwaZulu-Natal*
*SibandaP@ukzn.ac.za*

SAMS Classification: 23

The effects of vertical throughflow, rotation, cross-diffusion, and vertical heterogeneous permeability on the onset of double-diffusive convection in a finite rotating vertical porous cylinder have been studied. The fluid in the cylinder is warmed and salted from below with its top and lower walls considered to be isothermal, isosolutal, and permeable. In the model formulation, the Brinkman model was adopted, coupled with the Oberbeck-Boussinesq approximation. The normal mode technique is used to perform linear stability analysis and single term Galerkin method is used to solve the eigenvalue problem. Further, the influence of parameters such as thermal and solute Rayleigh, Taylor, and the Soret and Dufour numbers on the fluid system instability have been investigated. Among other results, we found that vertical heterogeneity may either stabilize or destabilize a fluid system. The stabilization of the throughflow remains consistent irrespective of the orientation in the absence of cross-diffusion effect. While the dual effect is observed in the presence of the cross-diffusion effect. We found that increasing the Dufour parameter delays the onset of both stationary and oscillatory convection. The positive Soret number has a stabilizing effect on the stationary convection case, and a destabilizing effect on the oscillatory convection case.

## A study comparing multidomain bivariate spectral quasilinearization method with block-hybrid method to solve nonlinear evolution equations

MAPULE PHEKO*

*North-West University*
*29298679@mynwu.ac.za*

SICELO P. GOQO

*University of KwaZulu-Natal*
*goqos@ukzn.ac.za*

LETLHOGONOLO D. MOLELEKI

*North-West University*
*Letlhogonolo.Moleleki@nwu.ac.za*

SAMS Classification: 23

In the present study, spectral block-hybrid method [1-2] is employed for solving the second order nonlinear evolution partial differential eqautions (PDEs) that are specified over the enormous time intervals. We compare the spectral block-hybrid method with the results obtained from literature of the multidomain bivariate spectral quasilinearization method [3-4]. The chosen numerical experimentation include Burgers-Huxley equation, Burgers-Fisher equation, and Coupled Burgers' equation. By comparing numerical solutions to analytical solutions from the literature, the numerical solutions are proven for precision and effectiveness.

References:

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## Investigating the Impact of Multiple Feeding Attempts on Mosquito Dynamics via Mathematical Models

MARKDONAL GHAKANYUY BIME *

*University of Bamenda, Cameroon*
*bmarkdonal@gmail.com*

GIDEON A. NGWA

*University of Buea, Cameroon*

MIRANDA I. TEBOH-EWUNGKEM

*Lehigh University, USA*

KRISTAN A. SCHNEIDER

*University of Applied Sciences, Germany*

SAMS Classification: 10

A deterministic differential equation model for the dynamics of terrestrial forms of mosquito populations is studied. The model assesses the impact of multiple probing attempts by mosquitoes that quest for blood within human populations by including a waiting class for mosquitoes that failed a blood feeding attempt. The equations are derived based on the idea that the reproductive cycle of the mosquito can be viewed as a set of alternating egg laying and blood feeding outcomes realized on a directed path called the gonotrophic cycle pathway. There exists a threshold parameter, the basic offspring number for mosquitoes, whose nature is affected by the way we interpret the transitions involving the different classes on the gonotrophic cycle path. The trivial steady state for the system, which always exists, can be globally asymptomatically stable whenever the threshold parameter is less than unity. The non-trivial steady state, when it exists, is stable for a range of values of the threshold parameter but can also be driven to instability via a Hopf bifurcation. The model's output reveals that the waiting class mosquitoes do contribute positively to sustain mosquito populations as well as increase their interactions with humans via increased frequency and initial amplitude of oscillations. A nonlinear analysis, based on the center manifold theory, is used to derive expressions for the amplitude and phase of the oscillating solutions. We conclude that to understand human-mosquito interactions, it is informative to consider multiple probing attempts; known to occur when mosquitoes quest for blood meals within human populations.

## Joint Modelling of SPX Options, VIX Options, and VIX Futures using 2-Factor Models

NELSON KYAKUTWIKA*

*Stellenbosch University*
20349912*@sun.ac.za*

BRUCE BARTLETT

*Stellenbosch University*
*bbartlett@sun.ac.za*

MESIAS ALFEUS

*Stellenbosch University*
*mesias@sun.ac.za*

ERIK SCHLOGL

*University of Technology Sydney, Australia*
*erik.schlogl@uts.edu.au*

SAMS Classification: 12

The joint modelling of SPX options, VIX options, and VIX futures is a unique challenge in the financial mathematics literature. Since 2006, significant progress has been made towards building models that jointly and accurately calibrate to the said SPX-VIX derivatives. However, only a few of these models have been capable of effective hedging in the SPX-VIX market. Building upon the (one-factor) Quintic Ornstein-Uhlenbeck Volatility Model proposed by Abi Jaber et al. (2022), we develop an enhanced model. This new model employs two Brownian motions to drive the volatility process, akin to the two-factor Bergomi model. Our results show that, with a few additional model parameters, our two-factor model gives an improved joint fit. We employ this calibrated model for hedging purposes in the SPX-VIX market.

## Coherent loop states and angular momentum

NZAGANYA NZAGANYA*

*Stellenbosch University*
*25177869@sun.ac.za*

BRUCE BARTLETT

*Stellenbosch University*
*bbartlett@sun.ac.za*

SAMS Classification: 20

We study Bohr-Sommerfeld states in the context of the irreducible representations of SU(2). These states offer a precise bridge between the classical and quantum descriptions of angular momentum. We show that they recover the usual basis of angular momentum eigenstates used in physics, and give a self-contained proof in this setting of the formula of Bothwick, Paul and Uribe [1] for the asymptotics of the inner product of arbitrary coherent loop states. As an application, we use these states to derive Littlejohn and Yu's [2] geometric formula for the asymptotics of the Wigner matrix elements.

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## How a gap analysis and mathematics can enhance service delivery at a local municipality in South Africa

ILSE SCHOEMAN*

*North-West University (Potchefstroom campus)*
*ilse.schoeman@nwu.ac.za*

JAMES CHAKWIZIRA

*North-West University (Potchefstroom campus)*
*Jameschakwizira@yahoo.com*

SAMS Classification: 12, 25

This talk will be based on the article [1]. The aim of the paper was to use forecasts (growth and backlog) in combination with baseline backlog weights to development a simplified performance management tool for intermediate and small municipalities in developing countries. This tool supports prioritization of projects to address the growing needs and service backlogs. Furthermore, Binary Optimization was used to show how to select the best combination of scenarios that maximized the total net present value given a limited budget and independent projects with different scenarios. Also, light was shed on data gaps and misalignment between policies and local service delivery needs.

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## Numerical Modelling of Blood Flow During Syringing

JAIMÉ GOEDHALS*

*Stellenbosch University*
*20893930@sun.ac.za*

ANDIE DE VILLIERS

*Stellenbosch University*
*andiedevilliers@sun.ac.za*

G. J. FRANCOIS SMIT

*Stellenbosch University*
*fsmit@sun.ac.za*

SAMS Classification: 23

Hemolysis, defined as the breaking open of red blood cells, can occur due to a range of factors which can be physical, chemical or biological in nature. Physical damage can manifest in scenarios such as emergency rapid blood transfusions conducted through syringing, a practice which may be employed in resuscitation procedures, particularly in rural hospitals. It has been demonstrated that such syringe-based transfusions result in noteworthy hemolysis, significantly surpassing the impact of pressure bag usage. This heightened hemolysis carries potential negative implications for the recipient of the transfusion. The core objective of this project revolves around emulating the blood flow through a sudden contraction, simulating a syringe, to emulate and quantify the underlying mechanisms driving hemolysis. Employing numerical simulations alongside a variety of hemolysis models, the goal is to identify the specific flow conditions that are most conducive to hemolysis occurrence. The modelling process commences with the depiction of blood flow as a Newtonian fluid traversing a sudden contraction, representing the syringe's action. This initial setup is subsequently expanded to account for non-Newtonian flow behaviours. To validate the modelling approach, a two-dimensional scenario is first simulated using the finite element software deal.II. This is then repeated and compared to results from numerical simulations using ANSYS Fluent. The scope extends further to encompass a three-dimensional setup, incorporating a representative syringe and hypodermic needle. The time history of shear stress along streamlines is then extracted to facilitate hemolysis analysis.

## Dynamics of an Asymmetric Wedge Billiard in a Constant Gravitational Field

KEEGAN ANDERSON*

*University of Johannesburg*
*kdanderson@uj.ac.za*

CHARLES VILLET

*University of Johannesburg*
*cmvillet@uj.ac.za*

SAMS Classification: 10

Mathematical billiards are considered to be a playground for mathematicians, because it combines techniques and methods of analysis from various different fields in mathematics. Billiard dynamics tend to be rich, depending on the shape of the billiard table and/or other modifications to the billiard system. For example, we may also consider the billiard ball moving within a potential field (gravitational or magnetic). The wedge billiard is mathematical billiard introduced and first studied by Lehtihet and Miller [1], where the billiard walls from a wedge shape at an angle and symmetrically about a vertical axis of symmetry. It was found that the dynamics are dependent on this angle the wedge makes with the vertical. In our research [2], we generalised this wedge to be asymmetric with respect to the vertical, and investigated the resulting dynamics. We were able to derive a condition which guarantees periodic orbits in the rotated orthogonal wedge billiard, which is a special case of the asymmetric wedge billiard [3].

References:

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## Propagation Properties of Internal Gravity Waves in a Stratified Fluid Over Topography

KHUTSANG KANAPI*

*University of the Witwatersrand*
*2305062@students.wits.ac.za*

SAMS Classification: 20

This study investigates the propagation properties of internal gravity wave in an inviscid and continuously stratified fluid over topography. We examine gravity waves in both rotating and non-rotating fluid systems without topography. Leveraging linear theory and the Boussinesq approximation, we derive the governing equations and utilize Fourier mode analysis to establish the dispersion relation. Our mathematical results, within an inertial reference frame, unveil the non-dispersive nature of these waves from which we we calculate the phase and group velocities. Furthermore, we establish that the wave vector is perpendicular to both the phase and group velocities, signifying the transverse nature of these waves. Additionally, the phase and group velocities are mutually perpendicular, while the wave frequency consistently remains lower than the buoyancy frequency. When considering the rotating frame we found that internal gravity waves are not affected by rotation. We then introduced the mean flow in the horizontal direction and calculated the group velocity in the vertical direction in terms of the mean flow. We found that when the mean flow is equal to the phase speed that is where we find the critical line and this is where dissipation occurs.

## Analysis of heat and mass transfer inside a combustion chamber

KATLEGO MOLAPO*

*North-West University*
*molapokatlego16@gmail.com*

GABRIEL MAGALAKWE

*North-West University*
*Gabriel.Magalakwe@nwu.ac.za*

LUCAS LEKOKO

*North-West University*
*Lucas.Lekoko@nwu.ac.za*

SAMS Classification: 23

The current research study investigates the heat, mass and concentration variation inside a combustion chamber and finds solutions that preserve the dynamics of the combustion process by predicting heat and mass transfer parameters which engineers can optimize or focus on to achieve better performance while improving overall system designs and performance. The model follows the conservation equations in the form of PDEs which are reduced to solvable ODEs through Lie's symmetry group of transformation [1]. The transformed equations are solved using successive linearization method on the Chebyshev nodes, see [2] and the perturbation technique is used to obtain numerical and semi-analytical solutions [3]. The solutions are used to study the effects of various parameters that influence the combustion process. Graphical representations of the mass and heat variation are presented and analysed.

References:

[1] Abel, M, Subhas, N, Mahesha,T and Jagadish T (2009). Heat transfer in a liquid film over an unsteady stretching surface with viscous dissipation in presence of external magnetic field, *Applied Mathematical Modelling*, **33** (8), 3430-3441.

[2] Kanzow, C, Nagel, C, Kato, H and Fukushima, M (2005). Successive linearization methods for nonlinear semidefinite programs, *Computational Optimization and Applications* **31**(3), 251-273.

[3] Abdelwahed, M, Hassine, M and Mohamed, M (2009). Optimal shape design for fluid flow using topological perturbation technique , *Journal of Mathematical Analysis and applications*, **356**(2), 548-563.

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